MATH 1005: College Algebra
Spring 2019 – April 12
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Answer Key
Quiz 9 – 3.6, 2.9, & 4.1
Total: 25 / 25

*Show **all** of your work in the space provided. Clearly indicate your final answer.*

1. Find the vertical asymptote(s), if any, of the graph of the following rational function $f(x) = \dfrac{(x+1)(2x-2)}{(x-3)(x+4)}$ 3 / 3

> **Solution:** The vertical asymptotes are the lines $x = a$ and $x = b$ where a and b are the numbers that make the denominator equal to zero, *after simplifying*. In this case, the expression cannot be simplified further, so the vertical asymptotes are $x = 3$ and $x = -4$.

2. Find the horizontal asymptote(s), if any, of the graph of the rational function $f(x) = \dfrac{10x}{7x + 6x^2}$ 2 / 2

> **Solution:** Since the highest power on x in the denominator is larger, the horizontal asymptote is the line $y = 0$.

3. Find the domain, range, and inverse function for the one-to-one function 5 / 5

$$f(x) = \frac{x+1}{x-2}.$$

> **Solution:** The domain of f is $(-\infty, 2) \cup (2, \infty)$. The range of f is the domain

of f^{-1}. So let's find f^{-1}:

$$\begin{aligned} y &= \frac{x+1}{x-2} \\ x &= \frac{y+1}{y-2} && \text{(swap } x \text{ and } y\text{)} \\ x(y-2) &= y+1 \\ xy-2x &= y+1 \\ xy-y &= 2x+1 \\ y(x-1) &= 2x+1 \\ y &= \frac{2x+1}{x-1} \end{aligned}$$

So $f^{-1}(x) = \dfrac{2x+1}{x-1}$. Since the domain of f^{-1} is $(-infty, 1) \cup (1, \infty)$, this is the range of f.

4. Let f be a one-to-one function given by $f(x) = 2x - 3$. If f^{-1} is the inverse function for f, what is $(f \circ f^{-1})(\pi)$? 4 / 4

Solution: By definition of the inverse function, we have $(f \circ f^{-1})(x) = x$. Let's think about what that means: When we first plug in a number x into f^{-1}, and then plug that output value into f, we just get x back. So $(f \circ f^{-1})(\pi) = \pi$. It actually didn't matter what $f(x)$ was.

5. Verify that the funtions $f(x) = 2x + 3$ and $g(x) = \dfrac{x-3}{2}$ are inverses of each other. 4 / 4

Solution:

$$\begin{aligned} f(g(x)) &= 2\left(\frac{x-3}{2}\right)+3 \\ &= (x-3)+3 \\ &= x \\ g(f(x)) &= \frac{(2x+3)-3}{2} \\ &= \frac{2x}{2} \\ &= x. \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions f and g are inverse to one another.

6. Find the exponential function of the form $f(x) = c \cdot a^x$ whose graph contains the points $(0, 3)$ and $(2, 12)$. 3 / 3

Solution: We have

$$\begin{aligned} 3 &= f(0) = ca^0 = c \cdot 1 = c \\ 12 &= f(2) = ca^2. \end{aligned}$$

Since the first equation gives $c = 3$, we can plug this into the second equation to find a:

$$\begin{aligned} 12 &= (3)a^2 \\ 4 &= a^2 \\ a &\pm 2. \end{aligned}$$

Since we must always have $a > 0$ in exponential functions, we take $a = 2$. So $f(x) = 3 \cdot 2^x$.

7. The U.S. population was 308 million in 2010. The U.S. Census Bureau reported that the population grew exponentially by 9.7% since 2000. What was the population of the U.S. in 2000? 4 / 4

Solution: I forgot the word "exponentially" in the statement of the problem on the actual quiz. But no one asked me about this during the quiz, and I think it's safe to say that everyone assumed it should be there. We have an exponential growth model function

$$A(t) = A_0 e^{kt},$$

where A_0 is the initial (or principal) amount at time $t = 0$. So $A(0) = A_0 = 308$. This means that the year 2010 is considered time $t = 0$. Our k is the rate of growth, which is $k = .097$. So the population in the year 2000 corresponds to time $t = -10$, so

$$A(-10) = 308e^{(.097)(-10)}.$$