MATH 1005: College Algebra
Spring 2019 – April 5
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Answer Key
Quiz 8 – 3.3, 3.4, & 3.5
Total: 25 / 25

*Show **all** of your work in the space provided. Clearly indicate your final answer.*

1. Use the factor theorem to determine if $x + 6$ is a factor of $x^3 + 9x^2 + 14x - 24$. 3 / 3

Solution: The factor theorem says that $x+6$ is a factor of $x^3+9x^2+14x-24$ if and only if -6 is a zero of the polynomial $x^3 + 9x^2 + 14x - 24$. So

$$(-6)^3 + 9(-6)^2 + 14(-6) - 24 = -216 + 324 - 84 - 24 = 0.$$

Hence $x + 6$ *is* a factor of $x^3 + 9x^2 + 14x - 24$. Alternatively, being a zero of the polynomial is equivalent to saying that $\frac{x^3+9x^2+14x-24}{x+6}$ has remainder zero. So you can also use long division and see if there is a remainder of zero. However, since $x + 6$ is of the form $x - a$, we can use synthetic division:

	1	9	14	− 24
− 6		− 6	− 18	24
	1	3	− 4	0

So we see that $\frac{x^3+9x^2+14x-24}{x+6}$ has remainder zero, which shows showing another way that $x + 6$ is indeed a factor.

2. Find the set of possible rational zeros of the function $f(x) = 5x^3 - 5x^2 + 2$. 3 / 3

Solution: This is a direct aplication of the remainder theorem: The possible rational zeros of f are

$$\pm 1, \pm 2, \pm\frac{1}{5}, \pm\frac{2}{5}.$$

3. A degree 3 polynomial $f(x)$ has zeros -5 and $2 - i$. Find the polynomial $f(x)$. 4 / 4

Solution: The fact that f has degree 3 tells us that there should be three zeros of f. By the conjugate pairs theorem, since $2-i$ is a zero of f, then $2+i$ must also be a zero of f. Moreover, a number a being a zero of f is equivalent to $x-a$ being a factor of f. Hence

$$\begin{aligned} f(x) &= (x-(-5))(x-(2-i))(x-(2+i)) \\ &= (x+5)((x-2)+i)((x-2)-i) \\ &= (x+5)((x-2)^2-i^2) \\ &= (x+5)(x^2-4x+5) \\ &= x^3+x^2-15x+25. \end{aligned}$$

4. Find the remaining zeros of a degree 5 polynomial with zeros $-1, 3+7i$, and $-2i$. 3 / 3

Solution: The fact that the polynomial has degreee 5 tells us that it will have 5 total zeros. Since $3+7i$ and $-2i$ are complex zeros, then we must also have $3-7i$ and $2i$ as zeros as well, by the conjugate pairs theorem. This gives us all 5 zeros.

5. Find all the zeros of the polynomial x^3-5x^2+9x-9. (Hint: You'll need the quadratic formula.) 6 / 6

Solution: The goal of this problem is to factor the given polynomial completely. Then, all of the linear factors will give us the zeros of the polynomial, because once again: A number a is a zero of a polynomial if and only if $x-a$ is a factor of the polynomial. So to find the zeros, we need to find all of the factors. Since our polynomial has degree 3, we expect that there are 3 total zeros.

Using the rational zeros theorem, we find that all possible rational zeros of the polynomial are: $\pm1, \pm3, \pm9$. Using synthetic division, we find that 3 is a zero of the polynomial.

$$\begin{array}{r|rrrr} & 1 & -5 & 9 & -9 \\ 3 & & 3 & -6 & 9 \\ \hline & 1 & -2 & 3 & 0 \end{array}.$$

This means that

$$x^3 - 5x^2 + 9x - 9 = (x-3)(x^2 - 2x + 3).$$

Now we need to factor $x^2 - 2x + 3$. To do this, we will find its zeros by using the quadratic formula:

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2i\sqrt{2}}{2} = 1 \pm i\sqrt{2}.$$

Hence

$$x^3 - 5x^2 + 9x - 9 = (x-3)(x - (1 + i\sqrt{2}))(x - (1 - i\sqrt{2})),$$

meaning that $x^3 - 5x^2 + 9x - 9$ has zeros: $3, 1 + i\sqrt{2}$, and $1 - i\sqrt{2}$.

6. Given that $3i$ is a zero of the polynomial $P(x) = x^3 - 2x^2 + 9x - 18$, find the remaining zeros of P. 6 / 6

Solution: Since $3i$ is a complex zero of $x^3 - 2x^2 + 9x - 18$, then $-3i$ is also a zero of the polynomial. Hence we know

$$x^3 - 2x^2 + 9x - 18 = (x - 3i)(x - (-3i))(\text{somethin' else}).$$

Then

$$(x - 3i)(x + 3i) = x^2 - 9i^2 = x^2 + 9.$$

Finally, we have

$$\begin{array}{r|rrrr}
 & & & x & -2 \\ \hline
x^2 + 9 & x^3 & -2x^2 & +9x & -18 \\
 & -x^3 & & -9x & \\ \hline
 & & -2x^2 & & -18 \\
 & & 2x^2 & & +18 \\ \hline
 & & & & 0
\end{array}$$

and hence

$$x^3 - 2x^2 + 9x - 18 = (x - 3i)(x + 3i)(x - 2),$$

which means $x^3 - 2x^2 + 9x - 18$ has zeros $2, 3i$, and $-3i$.