

Show **all** of your work in the space provided. Clearly indicate your final answer.

1. Let  $f(x) = \frac{5}{x}$  and  $g(x) = \frac{5}{x+1}$ . Find  $(f \circ g)(x)$ , and find the domain of  $f \circ g$ . 4 / 4

**Solution:**

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{5}{x+1}\right) = \frac{5}{\frac{5}{x+1}} = \frac{5}{1} \cdot \frac{x+1}{5} = x+1.$$

The domain of the function  $f \circ g$  is the domain of  $g$  intersected with the domain of  $x+1$ :

$$\begin{aligned}(\text{domain of } g) \cap (\text{domain of } x+1) &= ((-\infty, -1) \cup (-1, \infty)) \cap (-\infty, \infty) \\ &= (-\infty, -1) \cup (-1, \infty)\end{aligned}$$

2. Express the function  $H$  as the composition of two functions  $f$  and  $g$  such that  $H(x) = (f \circ g)(x)$ , where  $H(x) = \sqrt{x+7}$ . 3 / 3

**Solution:** Let  $f(x) = \sqrt{x}$  and let  $g(x) = x+7$ . Then

$$H(x) = (f \circ g)(x) = f(g(x)) = f(x+7) = \sqrt{x+7}.$$

3. Find the quadratic function  $f$  that has vertex  $(3, 4)$  and passes through the point  $(2, 6)$ . 3 / 3

**Solution:** The standard form of a quadratic equation is  $f(x) = a(x-h)^2+k$ , where  $(h, k)$  is the vertex. So

$$f(x) = a(x-3)^2+4.$$

Now we need to find  $a$ . Since we know that the point  $(2, 6)$  is on the graph of the function, this means  $f(2) = 6$ . So

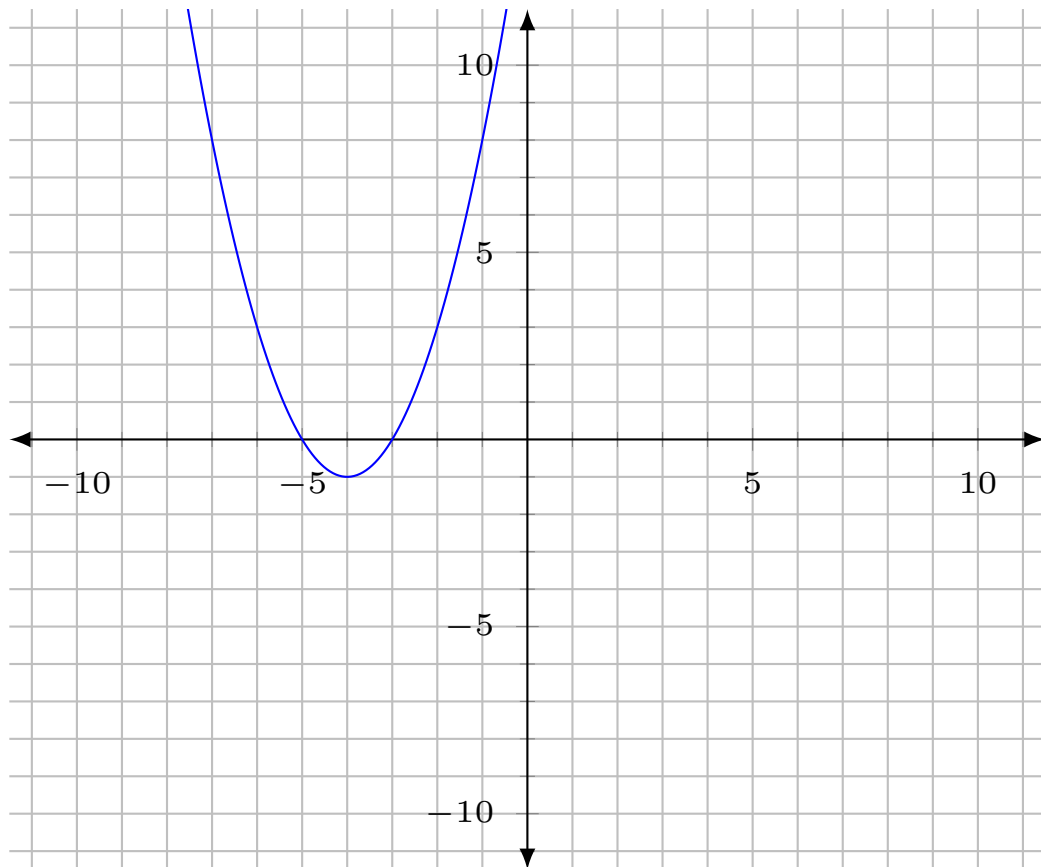
$$6 = f(2) = a(2-3)^2+4 = a(-1)^2+4 = a+4.$$

Hence  $a+4 = 6$ , with means  $a = 2$ . Therefore, we have  $f(x) = 2(x-3)^2+4$ .

4. Write the quadratic function  $f(x) = x^2 + 8x + 15$  in standard form, and then graph  $f(x)$  *without* using an  $xy$ -table. 5 / 5

**Solution:**

$$\begin{aligned} f(x) &= x^2 + 8x + 15 \\ &= x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 15 \\ &= x^2 + 8x + 16 - 16 + 15 \\ &= (x + 4)^2 - 16 + 15 \\ &= (x + 4)^2 - 1. \end{aligned}$$



5. Describe the end behavior of the polynomial function  $f$  given by  $f(x) = 4x - 2x^3$ . 3 / 3

**Solution:** The leading term of the polynomial is  $-2x^3$ . Hence the graph of  $f$  has end behavior “like” that of the function  $-x^3$ . So

$$\begin{aligned} f(x) &\rightarrow \infty \text{ as } x \rightarrow -\infty \\ f(x) &\rightarrow -\infty \text{ as } x \rightarrow \infty. \end{aligned}$$

6. Use long division to find the quotient and remainder:

4 / 4

$$\frac{32x^2 - 28x - 15}{8x + 3}$$

**Solution:**

$$\begin{array}{r} 4x - 5 \\ 8x + 3 \overline{) 32x^2 - 28x - 15} \\ \underline{- 32x^2 - 12x} \phantom{- 15} \\ - 40x - 15 \\ \underline{40x + 15} \\ 0 \end{array}$$

So,  $\frac{32x^2 - 28x - 15}{8x + 3} = 4x - 5$ .

7. Use synthetic division

3 / 3

$$\frac{2x^3 + 2x^2 - 3x + 5}{x + 4}$$

**Solution:**

$$\begin{array}{r|rrrr} -4 & 2 & 2 & -3 & 5 \\ & & -8 & 24 & -84 \\ \hline & 2 & -6 & 21 & -79 \end{array}$$

So,

$$\frac{2x^3 + 2x^2 - 3x + 5}{x + 4} = 2x^2 - 6x + 21 - \frac{79}{x + 4}.$$