$\boxed{1.}$ Simpson's Rule with $n=6$ to approximate $\int_0^{\pi} \cos(x^2)\,dx$.

Solution:

- $\Delta x = \frac{\pi - 0}{6} = \frac{\pi}{6}$
- $x_i = a + i\Delta x = 0 + i\left(\frac{\pi}{6}\right), \quad i = 0, 1, 2, \ldots, 6$
- $f(x) = \cos(x^2)$

$$\int_0^{\pi} \cos(x^2)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_5) + f(x_6)\right]$$

$$= \frac{\pi}{18}\left[\cos(0) + 4\cos\left(\left(\frac{\pi}{6}\right)^2\right) + 2\cos\left(\left(\frac{\pi}{3}\right)^2\right) + 4\cos\left(\left(\frac{\pi}{2}\right)^2\right) + 2\cos\left(\left(\frac{2\pi}{3}\right)^2\right) + 4\cos\left(\left(\frac{5\pi}{6}\right)^2\right) + \cos(\pi^2)\right]$$

$\boxed{2.}$ Suppose you know that $|f^{(4)}(x)| \leq 90$ on $[0,2]$. How large should n be so that the error, E_S, in the Simpson's Rule approximation is such that $|E_S| < 10^{-3}$?

Solution:

We know that $|E_S| \leq \frac{K(b-a)^5}{180n^4}$, where $|f^{(4)}(x)| \leq K$ for all x in $[a,b]$.

Therefore, $|E_s| \leq \dfrac{90(2-0)^5}{180n^4}$ since $|f^{(4)}(x)| \leq 90$

on $[0,2]$.

So, if we want $|E_s| \leq 10^{-3}$, then we need n

such that:

$$|E_s| \leq \frac{90(2)^5}{180n^4} \leq 10^{-3} = \frac{1}{1000}$$

$$\frac{1(32)}{2n^4} \leq \frac{1}{1000}$$

$$\frac{2n^4}{32} \geq 1000$$

$$n \geq \sqrt[4]{16000}.$$