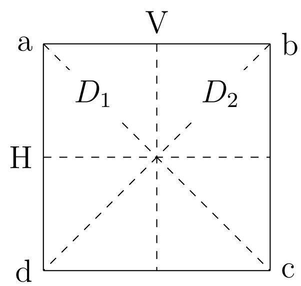
Intro to Abstract Algebra
Spring 2020 – February 25
Prof: Keiko Kawamuro – TA: Mr. Camacho

Answer Key
Quiz 4 – Section 8
Total: 20 / 20

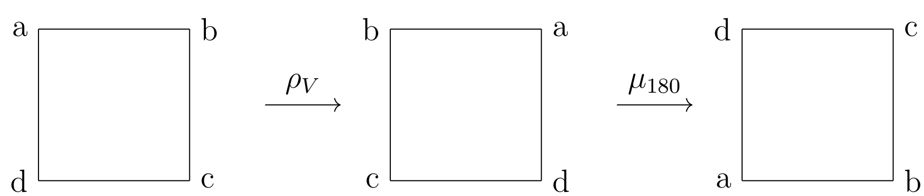
1. Consider the following square, with lines of symmetry V, H, D_1, and D_2: 5 / 5



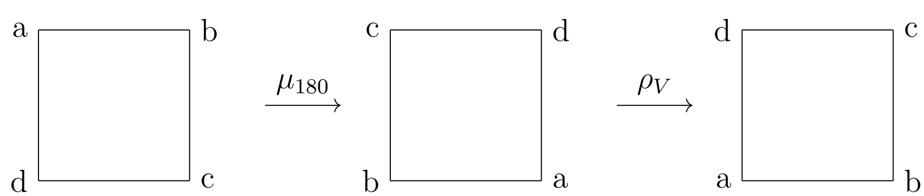
Note that μ_{180} denotes 180° clockwise rotation, and $\rho_V, \rho_H, \rho_{D_1}, \rho_{D_2}$ denote reflection through the various lines. Compute the following, and draw figures to verify.

Solution:

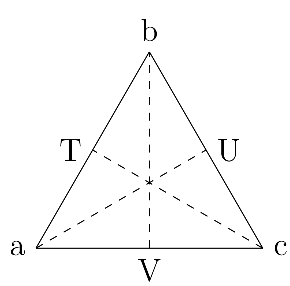
(a) $\mu_{180} \circ \rho_V = \rho_H$



(b) $\rho_V \circ \mu_{180} = \rho_H$



2. Determine the group of symmetries of the equilateral triangle: 10 / 10

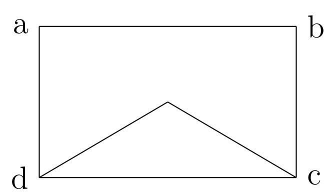


Solution:

Let ρ_T, ρ_U, and ρ_V denote reflection across the lines T, U, V, respectively. There are 3 rotations: $\mu_0, \mu_{120}, \mu_{240}$. Hence the group of symmetries of the equilateral triangle is a group with 6 elements with the following Cayley table:

$\circ$	μ_0	μ_{120}	μ_{240}	ρ_T	ρ_U	ρ_V
μ_0	μ_0	μ_{120}	μ_{240}	ρ_T	ρ_U	ρ_V
μ_{120}	μ_{120}	μ_{240}	μ_0	ρ_V	ρ_T	ρ_U
μ_{240}	μ_{240}	μ_0	μ_{120}	ρ_U	ρ_V	ρ_T
ρ_T	ρ_T	ρ_U	ρ_V	μ_0	μ_{120}	μ_{240}
ρ_U	ρ_U	ρ_V	ρ_T	μ_{240}	μ_0	μ_{120}
ρ_V	ρ_V	ρ_T	ρ_U	μ_{120}	μ_{240}	μ_0

3. Determine the group of symmetries of the following figure: 5 / 5



Solution:

The only rotation is the 0-degree rotation, μ_0, and the only line of symmetry is a vertical line in the middle of the rectangle. Call the reflection across this line ρ_V. Therefore, the group of symmetries of the figure is a group with two elements, with the following Cayley table:

$\circ$	μ_0	ρ_V
μ_0	μ_0	ρ_V
ρ_V	ρ_V	μ_0