

P.5 - Rational Expressions

- The quotient of two numbers is a rational number
- The quotient of two polynomials is a rational expression.

Ex: ① $\frac{2}{3}$ is a rational number

$\frac{x^2+7x-1}{x+3}$ is a rational expression.

② $\frac{(x+3)(x+2)}{x^2-7}$ is a rational expression.

• $(x+3)(x+2)$ is not in "standard form", i.e. it does not look like $ax^n + bx^m + cx^d + d$.

Reducing Rational Expressions to Lowest Terms

Steps: ① Factor the numerator and denominator.

② Cancel out any common factors (not term).

Ex: ① $\frac{x^4+2x^3}{x+2}, x \neq -2$

$$= \frac{x^3(x+2)}{x+2} = \frac{x^3}{1} = x^3$$

$$\textcircled{2} \frac{x^2-x-6}{x^3-3x^2} = \frac{(x-3)(x+2)}{x^2(x-3)} = \frac{x+2}{x^2}$$

$$\textcircled{3} \frac{2x^3+8x^2}{3x^2+12x} = \frac{2x^2(x+4)}{3x(x+4)} = \frac{2x^2}{3x} = \frac{2x}{3} = \frac{2}{3}x$$

$$(4) \frac{x^2-4}{x^2+4x+4} = \frac{(x-2)(x+2)}{(x+2)(x+2)} = \frac{x-2}{x+2}$$

* Examples of Wrong Way to Cancel:*

~~$$(1) \frac{x^2}{x^2+2x+3} = \frac{1}{2x+3}$$~~

~~$$(2) \frac{(x+2)(x-5)}{3x^2+x+2} = \frac{x-5}{3x^2}$$~~

~~$$(3) \frac{x+3}{x+7} = \frac{3}{7}$$~~

Multiplication & Division of Rational Expressions

- Steps:
- ① Factor numerators & denominators of the fractions involved
 - ② Perform operation (i.e. multiply or divide)
 - ③ Cancel any common factors

Ex: ① $\frac{x^2+3x+2}{x^3+3x} \cdot \frac{2x^3+6x}{x^2+x-2} = \frac{(x+1)(x+2)}{x(x^2+3)} \cdot \frac{2x(x^2+3)}{(x+2)(x-1)}$

$$= \frac{2x(x+1)(x+2)(x^2+3)}{x(x^2+3)(x+2)(x-1)}$$

$$= \frac{2(x+1)}{x-1}$$

$$= \frac{2(x+1)}{x-1}$$

$$\textcircled{2} \quad \frac{3x^2 + 11x - 4}{8x^3 - 40x^2} = \frac{(3x-1)(x+4)}{8x^2(x-5)}$$

$$\frac{\frac{3x+12}{4x^4-20x^3}}{1} = \frac{\frac{3(x+4)}{4x^3(x-5)}}{1}$$

$$= \frac{(3x-1)(x+4)}{8x^2(x-5)} \cdot \frac{4x^3(x-5)}{3(x+4)}$$

$$= \frac{\cancel{4}x^{\cancel{3}}(x-5)(3x-1)\cancel{(x+4)}}{\cancel{2}4x^{\cancel{2}}(x-5)\cancel{(x+4)}}$$

$$= \frac{x(3x-1)}{6}$$

$$\textcircled{3} \quad \frac{x^2-9}{x^2-6x+9} \cdot \frac{5x-5}{x+3} = \frac{(x-3)(x+3)}{(x-3)(x-3)} \cdot \frac{5(x-1)}{x+3}$$

$$= \frac{5\cancel{(x-3)}(x-1)\cancel{(x+3)}}{(x-3)\cancel{(x-3)}\cancel{(x+3)}}$$

$$= \frac{5(x-1)}{x-3}$$

$$\textcircled{4} \quad \frac{\frac{x^2+2x-3}{x^2+8x+16}}{\frac{x-1}{3x+12}} = \frac{\frac{(x+3)(x-1)}{(x+4)(x+4)}}{\frac{x-1}{3(x+4)}}$$

$$= \frac{(x+3)(x-1)}{(x+4)(x+4)} \cdot \frac{3(x+4)}{x-1}$$

$$= \frac{3(x+3)\cancel{(x-1)}\cancel{(x+4)}}{(x+4)(x+4)\cancel{(x-1)}} = \frac{3(x+3)}{x+4}$$

Remark from Quiz 1:

$$~~2(x^2 y^{-3})^4 = 2^4 (x^2)^4 (y^{-3})^4~~$$

$$~~2(x^2 y^{-3})^4 = 2((x^2)^4 (y^{-3})^4)~~
~~= 2x^8 2y^{-12}~~$$

$$2(x^2 + y) = 2x^2 + 2y$$
$$2(x^2 y) = 2x^2 y$$

$$2(3^2 \cdot 4) = 2(9 \cdot 4) = 2(36) = 72$$

$$~~2(3^2 \cdot 4) = 2 \cdot 3^2 \cdot 2 \cdot 4 = 2 \cdot 9 \cdot 8 = 2 \cdot 72 = 144~~$$

Addition & Subtraction of Rational Expressions

$$\underline{\text{Ex:}} \quad \frac{3x-2}{x^2-5x+6} - \frac{2x-1}{x^2-5x+6} = \frac{3x-2 - (2x-1)}{x^2-5x+6}$$

$$= \frac{3x-2-2x+1}{x^2-5x+6}$$

$$= \frac{x-1}{x^2-5x+6}$$

$$= \frac{x-1}{(x-3)(x-2)}$$

Finding a LCD of Two Rational Expressions

Steps ① Factor denominators

② Form a product of the distinct factors that appear

③ Raise each factor from ② to the greatest power that appears in either denominator.

Ex: ① $\frac{3}{x^2-1} + \frac{x}{x^2+2x+1}$

Find LCD:

① $x^2-1 = (x-1)(x+1)$, $x^2+2x+1 = (x+1)(x+1) = (x+1)^2$

② $(x-1)(x+1)$

③ $(x-1)(x+1)^2$

$$\frac{3}{(x-1)(x+1)} \cdot \frac{(x+1)}{(x+1)} = \frac{3x+3}{(x-1)(x+1)^2}$$
$$\frac{x}{(x+1)^2} \cdot \frac{(x-1)}{(x-1)} = \frac{x^2-x}{(x-1)(x+1)^2}$$

} change each fraction

$$\frac{3}{(x-1)(x+1)} + \frac{x}{(x+1)^2} = \frac{3x+3}{(x-1)(x+1)^2} + \frac{x^2-x}{(x-1)(x+1)^2}$$
$$= \frac{x^2+2x+3}{(x-1)(x+1)^2}$$

$$\textcircled{2} \quad \frac{x+2}{x^2-x} - \frac{3x}{4(x-1)^2}$$

LCD

$$\textcircled{1} \quad x^2-x = x(x-1) \quad , \quad 4(x-1)^2$$

$$\textcircled{2} \quad 4x(x-1)$$

$$\textcircled{3} \quad 4x(x-1)^2$$

$$\frac{x+2}{x(x-1)} \cdot \frac{4(x-1)}{4(x-1)} = \frac{4(x-1)(x+2)}{4x(x-1)^2} = \frac{4(x^2+2x-x-2)}{4x(x-1)^2}$$

$$= \frac{4(x^2+x-2)}{4x(x-1)^2}$$

$$= \frac{4x^2+4x-8}{4x(x-1)^2}$$

$$\frac{x+2}{x(x-1)} - \frac{3x}{4x(x-1)^2} = \frac{4x^2+4x-8}{4x(x-1)^2} - \frac{3x}{4x(x-1)^2}$$

$$= \frac{4x^2+x-8}{4x(x-1)^2}$$

$$\textcircled{3} \quad \frac{4}{x^2-4x+4} + \frac{x}{x^2-4}$$

LCD

$$\textcircled{1} \quad x^2-4x+4 = (x-2)^2 \quad , \quad x^2-4 = (x-2)(x+2)$$

$$\textcircled{2} \quad (x-2)(x+2)$$

$$\textcircled{3} \quad (x-2)^2(x+2)$$

$$\frac{4}{(x-2)^2} \cdot \frac{(x+2)}{(x+2)} = \frac{4x+8}{(x-2)^2(x+2)}$$

$$\frac{x}{(x-2)(x+2)} \cdot \frac{(x-2)}{(x-2)} = \frac{x^2-2x}{(x-2)^2(x+2)}$$

$$\begin{aligned} \frac{4}{(x-2)^2} + \frac{x}{(x-2)(x+2)} &= \frac{4x+8}{(x-2)^2(x+2)} + \frac{x^2-2x}{(x-2)^2(x+2)} \\ &= \frac{x^2+2x+8}{(x-2)^2(x+2)} \end{aligned}$$