P.4 – Factoring Polynomials

- Factoring is the "opposite" of multiplying.

$$(x+3)(x-3) = x^2 - 9$$

"$(x-3)$ is a factor of x^2-9"

$\xrightarrow{\text{multiplying}}$

$\xleftarrow{\text{factoring}}$

* The example of numbers:

$$3 \cdot 2 \cdot 4 = 24$$

"3 is a factor of 24"

$\xrightarrow{\text{mult.}}$

$\xleftarrow{\text{factoring.}}$

Greatest Common Factor Method

① Look for a common factor in the coefficients

② Look for a common least exponent on x (or y) in each term.

Ex:

Polynomial	Common Factor	Factored Form
$18y+9$	9	$9(2y+1)$
$2y^2+6y$	$2y$	$2y(y+3)$
$7x^4+3x^3+x^2$	x^2	$x^2(7x^2+3x+1)$

$$\frac{7x^4}{x^2} + \frac{3x^3}{x^2} + \frac{x^2}{x^2}$$

$$7x^{4-2} = 7x^2$$

$5x+3$ none! $5x+3$

$x^2(x-3)+7(x-3)$ $x-3$ $(x-3)(x^2+7)$

Factoring Trinomials (Polynomials of the form ax^2+bx+c)

Method ①:

Ex: • $x^2+8x+15$

· Look for two numbers which multiply to 15 and add to 8.

1, 15 3, 5

−1, −15 −3, −5

So $x^2+8x+15 = (x+3)(x+5)$

• $x^2-6x-16$

1, −16 2, −8 4, −4

−1, 16 −2, 8

So $x^2-6x-16 = (x+2)(x-8)$

• $x^2-x-2 = (x-2)(x+1)$

• $x^2-3x-10 = (x-5)(x+2)$

Method ②: We will use the formulas:

$$A^2 + 2AB + B^2 = (A+B)^2$$
$$A^2 - 2AB + B^2 = (A-B)^2$$

Ex: • $x^2 + 4x + 4$

$A = x$ • Is the middle term twice the product
$B = 2$ of A and B, (i.e. x and 2)?

$$x^2 + 4x + 4 = (x+2)^2$$

• $9x^2 - 6x + 1$

• Are the first and last terms squares?

→ Yes! $3x \cdot 3x = 9x^2$, so $A = 3x$
$1 \cdot 1 = 1$, so $B = 1$

• Is $-6x = 2 \cdot A \cdot B$ or $-6x = (-2) \cdot A \cdot B$

$2 \cdot 3x \cdot 1 = 6x$ $\quad$ $-2 \cdot 3x \cdot 1 = -6x$

No! $\quad$ Yes!

So $9x^2 - 6x + 1 = (3x-1)^2$

Difference of Squares $\quad A^2 - B^2 = (A-B)(A+B)$

Ex: • $x^2 - 4$. $A = x$ & $B = 2$

So $x^2 - 4 = (x-2)(x+2)$

• $25x^2 - 49$. $A = 5x$, $B = 7$

So $25x^2 - 49 = (5x-7)(5x+7)$

Factor by Grouping

Ex: • $x^3 + 2x^2 + 3x + 6$

$= (x^3 + 2x^2) + (3x + 6)$

$= x^2(x+2) + 3(x+2)$ * common factor of $x+2$ *

$= (x+2)(x^2 + 3)$

• $x^3 + 3x^2 + x + 3 = x^2(x+3) + (x+3)$

$= (x+3)(x^2+1)$

• $x^2 + 4x + 4 - y^2 = (x^2+4x+4) - y^2$

$= (x+2)^2 - y^2$

$= [(x+2) - y]((x+2) + y)$

• $6x^3 - 3x^2 - 4x + 2 = 3x^2(2x-1) + (-2)(2x-1)$

$= (2x-1)(3x^2-2)$

• $12x^7 + 4x^5 + 3x^4 + x^2$

$= 4x^5(3x^2+1) + x^2(3x^2+1)$

$= (3x^2+1)(4x^5+x^2)$

Back to Factoring Trinomials:

① Try Method ① above:

Ex: $x^2 + x - 6 = (x+3)(x-2)$

② Go to Method ②

- Are the first and last terms squares?

  Ex: $36x^2 + 12x + 1$

  $36x^2 = (6x)^2$, so let $A = 6x$

  $1 = (1)^2$, so let $B = 1$

- Is $12x$ equal to $2 \cdot A \cdot B$ or $-2 \cdot A \cdot B$?

  $2 \cdot 6x \cdot 1 = 12x$

  So $36x^2 + 12x + 1 = (6x+1)^2$

③ Last Resort: Use trial & error

Ex: $6x^2 - 13x - 5 = (3x-5)(2x+1)$

$$\overset{\text{outter}}{(3x)(1)} + \overset{\text{inner}}{(-5)(2x)}$$

$$= 3x - 10x$$

$$= -7x$$

$(3x+5)(2x-1)$ ✗

$(3x+1)(2x-5)$ ✓