P.2 - Integer Exponents

- Given a real number a, and an integer n, the symbol a^n means a multiplied by itself n - times, i.e.

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n\text{-factors}}.$$

<u>Ex</u>: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$
$(-3)^2 = (-3) \cdot (-3) = 9$
$(5)^3 = 5 \cdot 5 \cdot 5 = 125$

<u>Zero Exponents</u>: any real number raised to the 0 power is 1, except 0.

<u>Ex</u>: $\pi^0 = 1$
$(-1.79)^0 = 1$ $\qquad 0^0 = 0.$
$(17)^0 = 1$

<u>Negative Exponents</u>: Let n be a positive integer. Then

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n$$

Ex: ① $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

② $(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} = -\frac{1}{64}$

③ $7^{-1} = \frac{1}{7^1} = \frac{1}{7}$

④ $\frac{1}{8^{-2}} = 8^2 = 64$

Careful! : $-2^2 \neq (-2)^2$

$-2^2 = -(2^2) = -4$

$(-2)^2 = (-2)\cdot(-2) = 4$

Product Rule for Exponents:

$$\boxed{a^m \cdot a^n = a^{m+n}}$$

Ex: ① $3^2 \cdot 3 = 3^3 = 27$

② $x^2 \cdot x^{52} = x^{54}$

③ $(-4y^2)(3y^7) = (-4)(3)(y^2)(y^7)$

$= -12y^9$

④ $2x^3 \cdot x^4 = 2x^7$

$x \neq 0$ ⑤ $x^7 \cdot x^{-7} = x^0 = 1$

⑥ $2^{-3} \cdot 2^4 = 2^1 = 2$

Quotient Rule for Exponents:

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

Ex: ① $\frac{5^{10}}{5^9} = 5^{10-9} = 5^1 = 5$

② $\frac{x^{-3}}{x^2} = x^{-3-2} = x^{-5} = \frac{1}{x^5}$

$\frac{1}{x^3 \cdot x^2} = \frac{1}{x^5}$

③ $\frac{3}{3^2} = 3^{1-2} = 3^{-1} = \frac{1}{3}$

④ $\frac{2x^3}{3x^{-4}} = \frac{2}{3} \cdot \frac{x^3}{x^{-4}} = \frac{2}{3} \cdot x^{3-(-4)} = \frac{2}{3}x^7$

$= \frac{2x^7}{3}$

Power-to-a-Power Rule:

~~$(2x^2)(x^3) = 2x^6$~~

$$\boxed{(a^m)^n = a^{m \cdot n}}$$

Ex: ① $(5^2)^0 = 5^{2 \cdot 0} = 5^0 = 1$

② $(x^3)^{-1} = x^{-3} = \frac{1}{x^3}$

③ $(-2^2)^2 = (-(2^2))^2 = (-4)^2 = 16$

$$\cancel{(-2^2)^2 = -2^4 = -(2^4) = -16}$$

④ $(x^{-2})^{-7} = x^{14}$

⑤ $((-3)^{-1})^3 = (-3)^{-1\cdot 3} = (-3)^{-3} = \dfrac{1}{(-3)^3} = \dfrac{1}{-27}$.

Interlude: Interval notation

① A number x is in the interval $[2, 3)$ if and only if $x \geq 2$ and $x < 3$, or

$$2 \leq x < 3$$

② x is in $[2, \infty)$ if and only if $x \geq 2$

③ x is in $(-\infty, 10)$ if and only if $x < 10$.

Power-of-a-Product Rule:

$$\boxed{(a \cdot b)^n = a^n \cdot b^n}$$

Careful!

$(a+b)^n \neq a^n + b^n$

Ex: ① $(3x)^2 = 3^2 \cdot x^2 = 9x^2$

② $(-3x^2)^2 = (-3)^2 \cdot (x^2)^2 = 9x^4$

③ $(xy)^{-4} = x^{-4}y^{-4} = \dfrac{1}{x^4} \cdot \dfrac{1}{y^4} = \dfrac{1}{x^4y^4}$

④ $(x^2y)^3 = (x^2)^3 y^3 = x^6y^3$.

Power-of-Quotient Rule:

$$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$$

Ex: ① $\left(\frac{7}{x}\right)^2 = \frac{7^2}{x^2} = \frac{49}{x^2}$

② $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

③ $\left(\frac{4}{3}\right)^{-3} = \frac{4^{-3}}{3^{-3}} = \frac{3^3}{4^3} = \frac{27}{64}$

④ $\left(\frac{2}{5}\right)^{-2} = \frac{2^{-2}}{5^{-2}} = \frac{5^2}{2^2} = \frac{25}{4}$.

Simplifying Exponential Expressions:

Ex: ① $(-4x^2y^3)(7x^3y) = -4 \cdot 7 \cdot x^2 \cdot x^3 y^3 y$

$= -28\, x^5 y^4$

② $\left(\frac{x^5}{2y^{-3}}\right)^{-3} = \frac{(x^5)^{-3}}{(2y^{-3})^{-3}} = \frac{x^{-15}}{2^{-3}(y^{-3})^{-3}} = \frac{x^{-15}}{2^{-3}y^9}$

$= \frac{2^3}{x^{15}y^9} = \frac{8}{x^{15}y^9}$

③ $\frac{5a^{-2}bc^2}{a^4b^{-3}c^2} = \frac{5}{1}\cdot\frac{a^{-2}}{a^4}\cdot\frac{b}{b^{-3}}\cdot\frac{c^2}{c^2}$

$= 5\cdot a^{-2-4}\cdot b^{1-(-3)}\cdot c^{2-2}$

$= 5a^{-6}b^4c^0$

$= \frac{5b^4}{a^6}$

④ $\frac{x^2(-y)^3}{(xy^2)^3} = \frac{x^2\cdot(-y^3)}{x^3y^6} = \frac{x^2}{x^3}\cdot -\frac{y^3}{y^6} = \frac{1}{x}\cdot -\frac{1}{y^3}$

$= -\frac{1}{xy^3}$

⑤ $\left(\frac{2xy}{x^2}\right)^3 = \frac{(2xy)^3}{(x^2)^3} = \frac{8x^3y^3}{x^6} = \frac{8y^3}{x^3}$

⑥ $\left(\frac{4x^{-2}}{xy^5}\right)^3 = \frac{(4x^{-2})^3}{(xy^5)^3} = \frac{4^3(x^{-2})^3}{x^3(y^5)^3} = \frac{64x^{-6}}{x^3y^{15}} = \frac{64x^{-6-3}}{y^{15}}$

$= \frac{64}{x^9y^{15}}$

⑦ $\frac{1}{x^3}\cdot\frac{(x^2)^3x^{-4}}{1}$

$= \frac{x^6\cdot x^{-4}}{x^3}$

$= \frac{x^2}{x^3}$

$= x^{2-3} = x^{-1} = \frac{1}{x}$

$\boxed{xyz^{-10} = \frac{xy}{z^{10}}}$

$-xy(-z)^{-10} = \frac{-xy}{(-z)^{10}}$

$= -\frac{xy}{z^{10}}$