Chapter P - Basic Concepts of Algebra

P.1 - The Real Numbers & Their Properties.

Def: • A **variable** is a letter used to represent an arbitrary or unknown number.

- A **constant** is a specific number.
- All positive & negative decimals (and 0) make up the **real numbers.**

-232, 4, 0, 17, -400,
- **Integers**: positive & negative whole numbers, and 0.

$\frac{2}{3}$, -14, $\frac{5}{11}$, $-4/3$, 1
- **Rational Numbers**: numbers that can be written as a ratio (or fraction).

$\pi = 3.1415\ldots$, $e = 2.7\ldots$, $\sqrt{2}$
- **Irrational Numbers**: numbers that cannot be written as a fraction.

week of
Starting 1/21 Math Lab
Hours: M 2:30-430
W: 3:30-430
Thursday 1/17 - 2:30-4:30

Union & Intersection

- Given any two sets A, B of real numbers, the **union** of A and B is the set of all numbers in A or B, or both, denoted $A \cup B$.

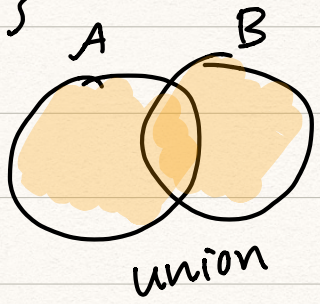
Ex $A = \{-1, 0, 1, 7, 14\}$

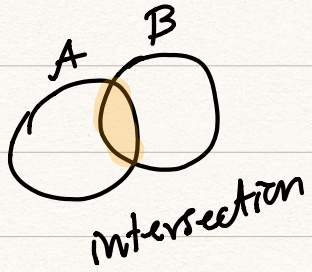
$B = \{-2, -1, 0, 3, 9\}$

$A \cup B = \{-2, -1, 0, 1, 3, 7, 9, 14\}$

- Given two sets A, B of real numbers, the intersection of A and B is the set of all numbers in both A and B, denoted $A \cap B$.

Ex: $A \cap B = \{-1, 0\}$





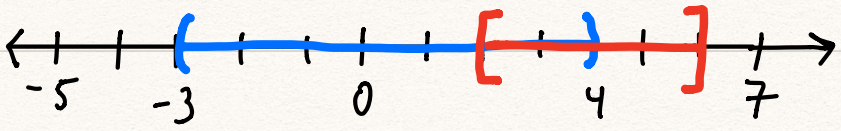
Intervals

- Suppose a, b are numbers with $a < b$. The open interval between a and b is the set of all real numbers between a and b, not including a and b, denoted (a, b).

  Ex: $a = 2$, $b = 5$. The open interval between 2 & 5, $(2, 5)$, is the set of all numbers between 2 & 5, not including 2 & 5.

- The closed interval between a & b is the same as the open interval (a, b), except we now include a & b, denoted $[a, b]$.
- The open interval (a, ∞) denotes all real numbers greater than a, not including a. Similarly, $(-\infty, a)$ is defined.
- Possible intervals: (a, b), $[a, b]$, $(a, b]$, $[a, b)$ (a, ∞), $[a, \infty)$, $(-\infty, b)$, $(-\infty, b]$, $(-\infty, \infty)$.

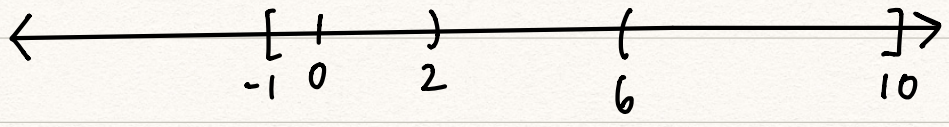
Ex: ① $(-3,4)$ & $[2,6]$



$(-3,4) \cup [2,6] = (-3,6]$

$(-3,4) \cap [2,6] = [2,4)$

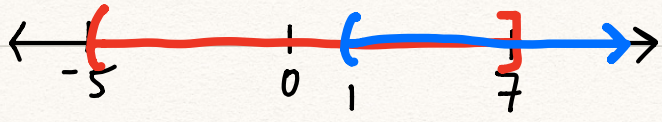
② $[-1,2)$ & $(6,10]$



$[-1,2) \cup (6,10]$

$[-1,2) \cap (6,10] = \varnothing$ = "empty set"

③ $(1,\infty)$ & $(-5,7]$



$(1,\infty) \cup (-5,7] = (-5,\infty)$

$(1,\infty) \cap (-5,7] = (1,7]$

Absolute Value

- Given a number a, the absolute value of a is its distance from 0, denoted $|a|$

Ex: $|-5| = 5$

$|1.978364| = 1.978364$

$|-14.\overline{24}| = 14.\overline{24}$

Order of Operation

① Parenthesis

② Exponents

③ [M / D]

④ [A / S]

Ex: $3 \cdot 2^5 + 4 = 3 \cdot 32 + 4$

$= 96 + 4$

$= 100$

$5 - (3-1)^2 = 5 - (2)^2$

$= 5 - 4$

$= 1$

Fractions:

Multiplication: multiply acrross the top & bottom

Ex: $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$

$5 \cdot \frac{10}{11} = \frac{5}{1} \cdot \frac{10}{11} = \frac{50}{11}$

Division: multiply the top by the reciprocal of the bottom.

Ex: $\frac{\frac{2}{3}}{\frac{7}{9}} = \frac{2}{3} \cdot \frac{9}{7} = \frac{18}{21} = \frac{6}{7}$

$\frac{\frac{4}{5}}{10} = \frac{4}{5} \cdot \frac{1}{10} = \frac{4}{50} = \frac{2}{25}$

Addition & Subtraction

Ex: ① $\frac{2}{3} + \frac{7}{4} = \frac{4}{4} \cdot \frac{2}{3} + \frac{3}{3} \cdot \frac{7}{4}$

$= \frac{8}{12} + \frac{21}{12}$

$= \frac{29}{12}$

② $\frac{4}{3} - \frac{7}{6} = \frac{2}{2} \cdot \frac{4}{3} - \frac{7}{6}$

$= \frac{8}{6} - \frac{7}{6}$

$= \frac{1}{6}$

or $\frac{4}{3} - \frac{7}{6} = \frac{6}{6} \cdot \frac{4}{3} - \frac{3}{3} \cdot \frac{7}{6}$

$= \frac{24}{18} - \frac{21}{18}$

$= \frac{3}{18}$

$= \frac{1}{6}$

③ $5 + \frac{5}{2} = \frac{5}{1} + \frac{5}{2}$

$$= \frac{2}{2} \cdot \frac{5}{1} + \frac{5}{2} \cdot \frac{1}{1}$$

$$= \frac{10}{2} + \frac{5}{2}$$

$$= \frac{15}{2}.$$