MATH 1005: College Algebra
Spring 2019 – February 14
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Answer Key
Midterm 1 – Chapter P; Sections 1.1, 1.2, & 1.3
Total: 70 / 70

*Show **all** of your work in the space provided. Clearly indicate your final answer.*

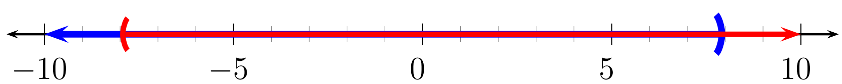
1. Evaluate $(-(-1)^5)(-2^4) + (\sqrt{2})^0$. 4 / 4

Solution: We have $(-1)^5 = -1$, $-2^4 = -(2^4) = -16$, and $(\sqrt{2})^0 = 1$, so

$$\begin{aligned}\left(-(-1)^5\right)\left(-2^4\right) + (\sqrt{2})^0 &= (-(-1))(-16) + 1\\ &= (1)(-16) + 1\\ &= -16 + 1\\ &= -15.\end{aligned}$$

2. Find the union, $A \cup B$, of the intervals $A = (-\infty, 8)$; $B = [-8, \infty)$. 3 / 3

Solution: Let's graph the interval A with blue, and the interval B with red:



The union of the two intervals includes all the numbers in *either* interval. In other words, the union must include everything that the blue graph *or* the red graph is touching. In this case, red or blue is touching everything, so we have $A \cup B = (-\infty, \infty)$.

3. Simplify $3\left(x^{-4}y^{10}z^2\right)^3$, leaving only positive exponents. 4 / 4

Solution:

$$\begin{aligned}3\left(x^{-4}y^{10}z^2\right)^3 &= 3\left(x^{-4}\right)^3\left(y^{10}\right)^3\left(z^2\right)^3\\ &= 3x^{-12}y^{30}z^6\\ &= \frac{3y^{30}z^6}{x^{12}}.\end{aligned}$$

4. Simplify $\left(\frac{x^{-2}y^3z^2}{y^4}\right)^{-2}$, leaving only positive exponents. 4 / 4

Solution:

$$\begin{aligned}\left(\frac{x^{-2}y^3z^2}{y^4}\right)^{-2} &= \frac{\left(x^{-2}\right)^{-2}\left(y^3\right)^{-2}\left(z^2\right)^{-2}}{\left(y^4\right)^{-2}} \\ &= \frac{x^4y^{-6}z^{-4}}{y^{-8}} \\ &= x^4y^{-6-(-8)}z^{-4} \\ &= \frac{x^4y^2}{z^4}.\end{aligned}$$

5. Evaluate $(5z+3)^2$. 3 / 3

Solution: Use the formula $(A+B)^2 = A^2 + 2AB + B^2$, or simply multiply the two polynomials as usual.

$$(5z+3)^2 = 25z^2 + 30z + 9.$$

6. Factor $4x^2 - 4x + 1$. 3 / 3

Solution: Notice that the first and last terms are squares, with $(2x)^2 = 4x^2$ and $1^2 = 1$. Letting $A = 2x$ and $B = 1$, we check if $-4x$ is equal to $2AB$ or $-2AB$. We see that

$$-2AB = -2(2x)(1) = -4x,$$

so we apply the formula $A^2 - 2AB + B^2 = (A-B)^2$ to obtain

$$4x^2 - 4x + 1 = (2x-1)^2.$$

7. Factor $x^2 - 53x - 54$. $3 \,/\, 3$

Solution: Since 1 and -54 are two numbers which multiply to -54 and add to -53, we have

$$x^2 - 53x - 54 = (x + 1)(x - 54)$$

8. Evaluate $\dfrac{(x-7)(x+3)}{(x^2-5x-14)} \cdot \dfrac{(x-2)(x+2)}{(x^2+x-6)}$. $3 \,/\, 3$

Solution: We first factor the numerator and denominator, multiply, and then cancel any common factors:

$$\frac{(x-7)(x+3)}{(x^2-5x-14)} \cdot \frac{(x-2)(x+2)}{(x^2+x-6)} = \frac{(x-7)(x+3)(x-2)(x+2)}{(x-7)(x+2)(x+3)(x-2)} = 1.$$

9. Simplify $\dfrac{x^2-9}{(x+3)(x-2)}$. $3 \,/\, 3$

Solution: We factor the numerator and denominator, then cancel out any common factors:

$$\frac{x^2-9}{(x+3)(x-2)} = \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-2)} = \frac{x-3}{x+2}.$$

10. Evaluate $\dfrac{2}{7} + \dfrac{3}{35}$. $2 \,/\, 2$

Solution:

$$\begin{aligned} \frac{2}{7} + \frac{3}{35} &= \frac{2}{7} \cdot \frac{5}{5} + \frac{3}{35} \\ &= \frac{10}{35} + \frac{3}{35} \\ &= \frac{13}{35}. \end{aligned}$$

11. Evaluate $\dfrac{x}{(2x+3)(2x-3)} + \dfrac{7x}{(2x+3)^2}$. 4 / 4

Solution: First we find an LCD. Our LCD in this case is $(2x-3)(2x+3)^2$. So

$$\begin{aligned}
\frac{x}{(2x+3)(2x-3)} + \frac{7x}{(2x+3)^2} &= \frac{x}{(2x-3)(2x+3)} \cdot \frac{2x+3}{2x+3} + \frac{7x}{(2x+3)^2} \cdot \frac{2x-3}{2x-3} \\
&= \frac{x(2x+3)}{(2x-3)(2x+3)^2} + \frac{7x(2x-3)}{(2x-3)(2x+3)^2} \\
&= \frac{2x^2+3x+14x^2-21x}{(2x-3)(2x+3)^2} \\
&= \frac{16x^2-18x}{(2x-3)(2x+3)^2} \\
&= \frac{2x(8x-9)}{(2x-3)(2x+3)^2}.
\end{aligned}$$

12. Evaluate $(-64)^{\frac{1}{3}}$, or state that it is not a real number. 2 / 2

Solution: We have $(-64)^{\frac{1}{3}} = \sqrt[3]{-64}$. Since our radicand is negative, we must have an odd index. Since 3 is odd, the expression is a real number. Since $(-4)^3 = -64$, then

$$(-64)^{\frac{1}{3}} = \sqrt[3]{-64} = -4.$$

13. Evaluate $(-16)^{\frac{1}{4}}$, or state that it is not a real number. 2 / 2

Solution: We have $(-16)^{\frac{1}{4}} = \sqrt[4]{-64}$. Since our radicand is negative, we must have an odd index. But our index, 4, is even, so the expression is *not* a real number.

14. Simplify $\sqrt{45x} + \sqrt{20x}$. 3 / 3

Solution: To simplify $\sqrt{45x}$, we ask if either 45 or x are perfect squares, or if they are divisible by perfect squares. Although 45 is not a perfect square, it is divisible by the perfect square 9, (and x is neither a perfect square not divisible by a perfect square). Similarly for $\sqrt{20x}$, we obtain

$$\begin{aligned}\sqrt{45x} + \sqrt{20x} &= \sqrt{9 \cdot 5 \cdot x} + \sqrt{4 \cdot 5 \cdot x} \\ &= \sqrt{9}\sqrt{5 \cdot x} + \sqrt{4}\sqrt{5 \cdot x} \\ &= 3\sqrt{5x} + 2\sqrt{5x} \\ &= 5\sqrt{5x}.\end{aligned}$$

15. Rationalize the denominator in the expression $\dfrac{\sqrt{5}}{\sqrt{3}}$, i.e. change the expression so that no radical appears in the denominator. 2 / 2

Solution:

$$\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{5 \cdot 3}}{\sqrt{3 \cdot 3}} = \frac{\sqrt{15}}{3}.$$

16. Solve for x: $2x - 3 - (3x - 1) = 6$. 2 / 2

Solution:

$$\begin{aligned}2x - 3 - (3x - 1) &= 6 \\ 2x - 3 - 3x + 1 &= 6 \\ -x - 2 &= 6 \\ -x &= 8 \\ x &= -8\end{aligned}$$

17. Solve for ℓ in the equation $P = 2w + 2\ell$, (the perimeter of a rectangle). 3 / 3

Solution:

$$
\begin{aligned}
P &= 2w + 2\ell \\
P - 2w &= 2\ell \\
\frac{P - 2w}{2} &= \ell.
\end{aligned}
$$

18. A computer is on sale for 30% off the original price. If the sale price is \$805, what is the original price of the computer? 5 / 5

Solution: Let x=original price of the computer. Now, "%30 off" means that we subtract $.3x$ from x. So our equation modeling this problem is $x - .3x = 805$. Solving for x,

$$
\begin{aligned}
x - .3x &= 805 \\
x(1 - .3) &= 805 \\
x(.7) &= 805 \\
x\left(\frac{7}{10}\right) &= 805 \\
7x &= 8050 \\
x &= 1150.
\end{aligned}
$$

19. The length of a rectangle is 3 feet less than twice its width. The perimeter of the rectangle is 24 feet. Find the length and width of the rectangle. 5 / 5

Solution: Let w be the width of the rectangle, and ℓ be the length of the rectangle. We are told that ℓ is 3 feet less than twice of w, which means $\ell = 2w - 3$. We also know that the perimeter of the rectangle is 24, so

$$24 = \ell + \ell + w + w = 2\ell + 2w.$$

Substituting $2w - 3$ for ℓ, we get

$$\begin{aligned} 24 &= 2(2w - 3) + 2w \\ &= 4w - 6 + 2w \\ &= 6w - 6. \end{aligned}$$

So $24 = 6w - 6$, which means $30 = 6w$, hence $w = 5$. Then

$$\ell = 2w - 3 = 2(5) - 3 = 7.$$

20. Solve $3x^2 = 5x + 2$ by using the quadratic formula. $5 \,/\, 5$

Solution: First, let's change the equation so that we have quadratic in standard form on the left hand side:

$$\begin{aligned} 3x^2 &= 5x + 2 \\ 3x^2 - 5x - 2 &= 0 \end{aligned}$$

The quadratic on the left hand side is not easily factored, and if we want to complete the square, we'd have to divide the equation by 3, which is less than ideal. So, we use the quadaratic formula to find x:

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} \\ &= \frac{5 \pm \sqrt{25 + 24}}{6} \\ &= \frac{5 \pm 7}{6}. \end{aligned}$$

So $x = 2$ or $x = -\frac{1}{3}$.

21. Solve $x^2 + 2x - 5 = 0$ by completing the square. $5 \,/\, 5$

Solution: To complete the square, we make sure that the x^2 has a coefficient of 1 (which in this case it already does), and that the constants are on the opposite side of the equation than those terms with x. Then we add the square of half the coefficient of x to both sides.

$$\begin{aligned} x^2 + 2x - 5 &= 0 \\ x^2 + 2x &= 5 \\ x^2 + 2x + 1 &= 5 + 1 \\ (x+1)^2 &= 6 \\ x + 1 &= \pm\sqrt{6} \\ x &= -1 \pm \sqrt{6} \end{aligned}$$