MATH 1005: College Algebra
Spring 2019 – March 14
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Answer Key
Midterm 2 – Sections 1.4 to 1.6 & 2.1 to 2.8
Total: 67 / 67

*Show **all** of your work in the space provided. Clearly indicate your final answer.*

1. Use the quadratic formula to solve the quadratic equation $x^2 - 6x + 10 = 0$. 4 / 4

Solution:

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i.$$

2. Solve the equation $|x+1| - 7 = -2$ 4 / 4

Solution:

$$|x+1| - 7 = -2$$
$$|x+1| = 5$$
$$x + 1 = 5 \text{ or } x + 1 = -5$$
$$x = 4 \text{ or } x = -6$$

3. Solve the inequality $|2x+1| + 3 \leq 8$. 4 / 4

Solution: We first solve for the abolustion value expression, and then break it up into an AND statement, since the inequality is a less "THAND":

$$|2x+1| + 3 \leq 8$$
$$|2x+1| \leq 5$$
$$2x + 1 \leq 5 \text{ and } 2x + 1 \geq -5$$
$$2x \leq 4 \text{ and } 2x \geq -6$$
$$x \leq 2 \text{ and } x \geq -3.$$

This was an acceptable answer. However, if we were asked to give our final answer in interval notation, the final answer would be:

$$(-\infty, 2] \cap [-3, \infty) = [-3, 2].$$

4. Solve the rational inequality using the test point method. Write you final answer in interval notation. 5 / 5

$$\frac{(x+1)(x+2)}{x-5} \geq 0$$

Solution: Typing up a solution to this problem is a bit difficult. See lecture notes for section 1.6 for detailed examples. The final answer for this problem is:

$$[-2,-1] \cup (5,\infty).$$

5. Consider the points $P=(2,7)$ and $Q=(-3,6)$. 6 / 6

Solution:

(a) Find the distance between P and Q.

$$d(P,Q) = \sqrt{(-3-2)^2+(6-7)^2} = \sqrt{26}.$$

(b) Find the midpoint between P and Q.

$$\left(\frac{2+(-3)}{2}, \frac{7+6}{2}\right) = \left(-\frac{1}{2}, \frac{13}{2}\right).$$

(c) Find the slope of the line between P and Q.

$$m = \frac{6-7}{-3-2} = \frac{-1}{-5} = \frac{1}{5}.$$

6. Find the center and radius of the circle given by the equation 4 / 4

$$x^2+y^2+6x-10y-6=0$$

Solution:

$$\begin{aligned} x^2 + y^2 + 6x - 10y &= 6 \\ x^2 + 6x + y^2 - 10y &= 6 \\ x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 &= 6 + \left(\frac{6}{2}\right)^2 + \left(\frac{-10}{2}\right)^2 \\ x^2 + 6x + 9 + y^2 - 10y + 25 &= 6 + 9 + 25 \\ (x+3)^2 + (y-5)^2 &= 40 \end{aligned}$$

So the circle has center $(-3, 5)$ and radius $\sqrt{40} = 2\sqrt{10}$.

7. Give the equation of the line passing through the point $(-1, 3)$ and parallel to the line $y = 3x + 1$. 3 / 3

Solution: Since our line is parallel to a line with slope 3, our line has slope 3. So our line is given by $y = 3x + b$. Now we need to find b. Since our line passes through $(-1, 3)$, we can plug in these values to our equation to find b:

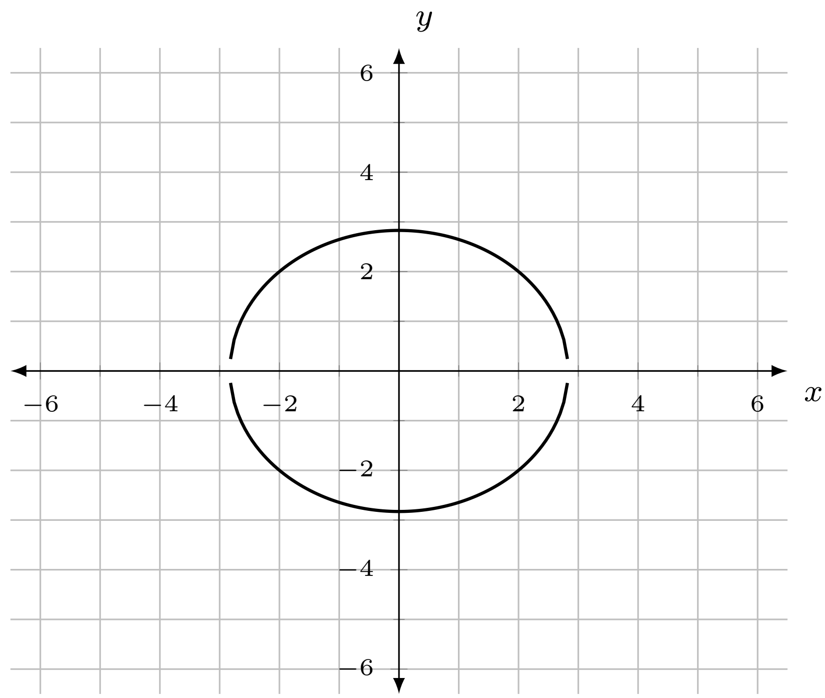
$$\begin{aligned} y &= 3x + b \\ 3 &= 3(-1) + b \\ b &= 6 \end{aligned}$$

So our line is given by $y = 3x + 6$.

8. Find the domain of the function $g(x) = \dfrac{x}{x-1}$. 3 / 3

Solution: The domain of a function is all the points we are allowed to plug in. In this case, that includes all the numbers exxcept the one that makes the denominator equal to 0. So our domain is $(\infty, 1) \cup (1, \infty)$.

9. Does the following graph depict the graph of a function? Explain. 3 / 3



Solution: No! The graph does not pass the vertical line test. In other words, therre are mulitple output values for a given input value.

10. Determine algebraically whether the function f given by $f(x) = 3x^3 + 2x + 7$ is odd, even, or neither. Do the same for the function g given by $g(x) = \dfrac{x^2}{x-2}$. 3 / 3

Solution:

$$f(-x) = 3(-x)^3 + 2(-x) + 7 = -3x^3 - 2x + 7$$
$$g(-x) = \frac{(-x)^2}{(-x)-2} = \frac{x^2}{-x-2}.$$

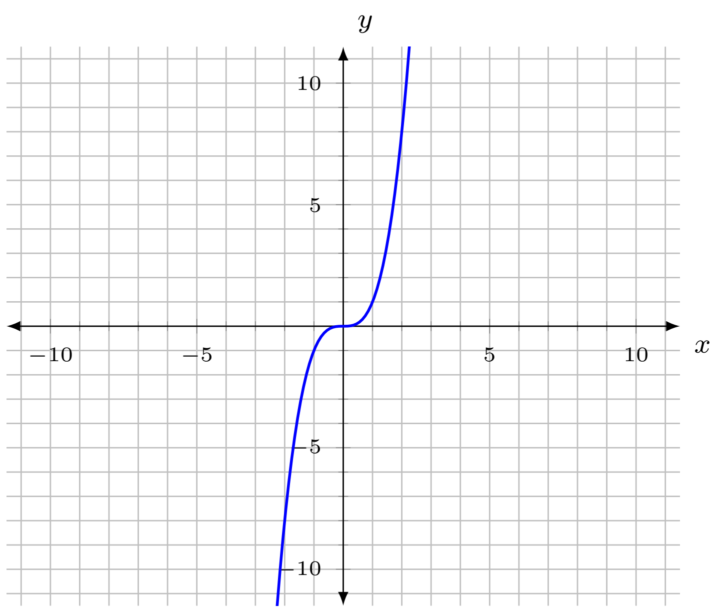
Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, the function f is neither even nor odd. For the same reasons, g is neither even nor odd.

11. Draw the graphs of the following functions without using and xy-table. Also determine geometrically if the functions are even, odd, or neither. 4 / 4

Solution:

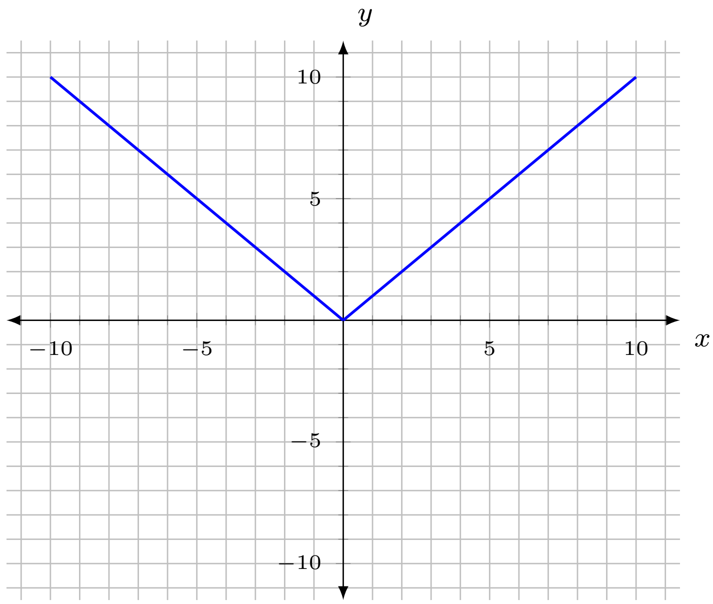
(a) $f(x) = x^3$

x^3 is an odd function, since it is symmetric about the origin.



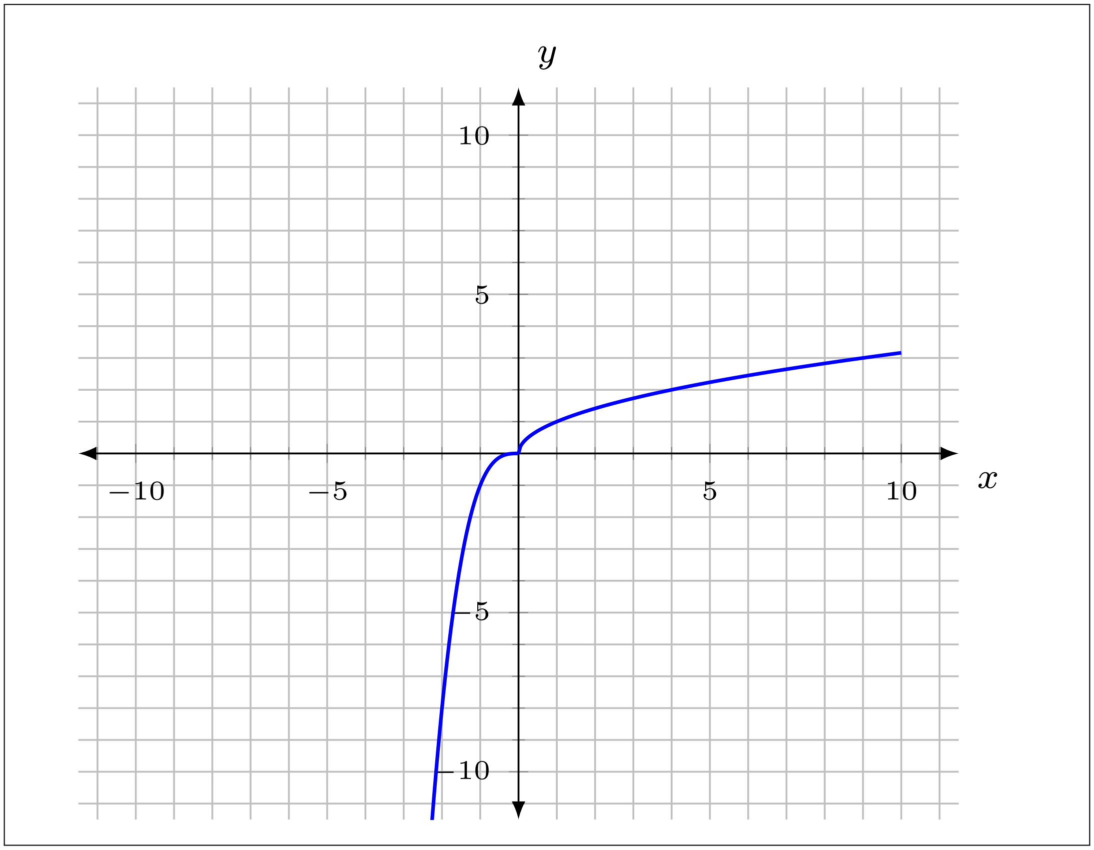
(b) $g(x) = |x|$

$|x|$ is an even function, since it is symmetric about the y-axis.



12. Graph the piecewise function: $f(x) = \begin{cases} x^3 & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$. 4 / 4

Solution:



13. The function f given by the rule $f(x) = 2\sqrt[3]{x+2} - 1$ is a transformation of a "standard function". Indicate what this standard function is, and the transformations needed to obtain $f(x)$. 4 / 4

Solution: The standard function is $\sqrt[3]{x}$. The transofmrations needed to obtain f from $\sqrt[3]{x}$ are:

1. Shift left 2
2. Vertical stretch by a factor of 2.
3. Shift down 1.

14. Write an equation for a function whose graph fits the given description: The graph of $g(x) = \sqrt{x}$ is shifted left 4 units, reflected across the x-axis, and compressed horizontally by a factor of 3. 4 / 4

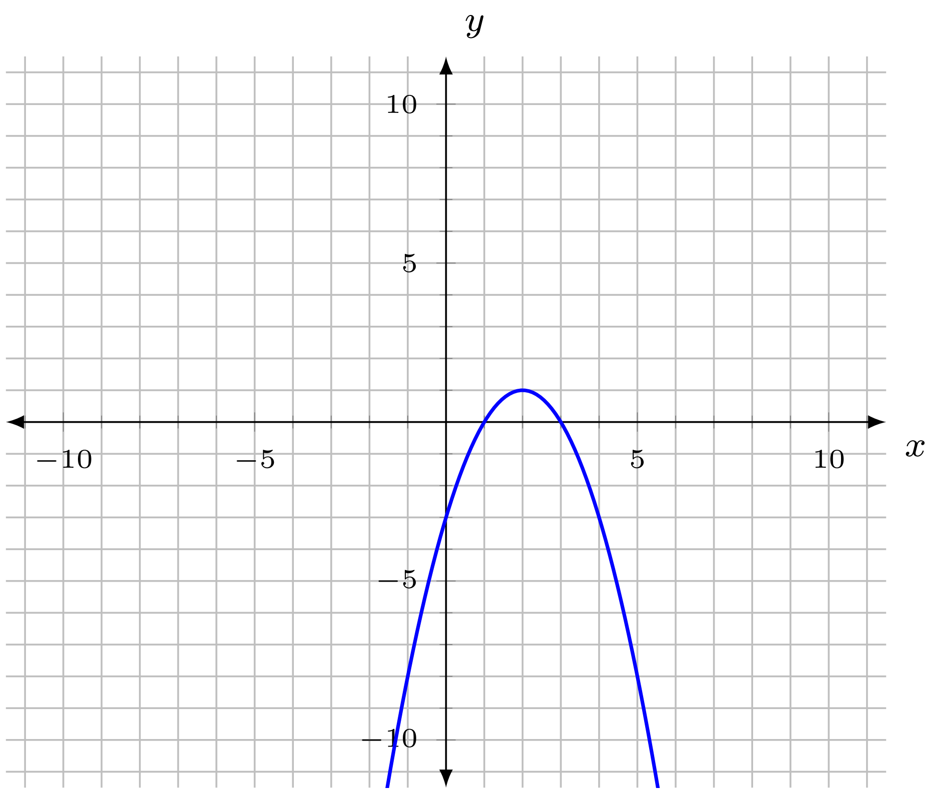
Solution:

$$\begin{aligned} y &= \sqrt{x} && \\ y &= \sqrt{x+4} && \text{(shift left 4 units)} \\ -y &= \sqrt{x+4} && \text{(reflect across } x\text{-axis)} \\ y &= -\sqrt{x+4} && \text{(solving for } y\text{)} \\ y &= -\sqrt{3x+4}. && \text{(horizontal compression by factor of 3)} \end{aligned}$$

So $f(x) = -\sqrt{3x+4}$ is the desired function.

15. Graph the function obtained by shifting the graph of $f(x) = x^2$ by 2 units to the right, reflected across the x-axis, and shifted up 1 unit. 4 / 4

Solution:



16. Let $f(x) = \dfrac{x^2+5}{x-1}$ and let $g(x) = \sqrt{x}$. Find the composite function $(g \circ f)(x)$ and determine the domain of $(g \circ f)(x)$. 4 / 4

Solution:

$$(g \circ f)(x) = g(f(x)) = \sqrt{\frac{x^2+5}{x-1}}.$$

So domain of $g \circ f$ is the domain of f intersected with the domain of $\sqrt{\frac{x^2+5}{x-1}}$. The domain of f is $(-\infty, 1) \cup (1, \infty)$. The domain of $\sqrt{\frac{x^2+5}{x-1}}$ is wherever

$$\frac{x^2+5}{x-1} \geq 0.$$

Using the test point method to solve this inequality, we find that the domain of $\sqrt{\frac{x^2+5}{x-1}}$ is $(1, \infty)$. So the domain of $g \circ f$ is

$$((-\infty, 1) \cup (1, \infty)) \cap (1, \infty) = (1, \infty).$$

17. The function $h(x) = (x-4)^2 + 5$ is a composition of functions. Determine the two functions f, g such that $h(x) = (f \circ g)(x)$ 4 / 4

Solution: Let $f(x) = x^2 + 5$ and $g(x) = x - 4$. Then

$$(f \circ g)(x) = f(g(x)) = (x-4)^2 + 5 = h(x),$$

as desired.