Solutions

College Algebra
Lots of Logs
Mr. Camacho

1. Write the following in exponential form.

a) $\log_3 81 = 4$

$3^4 = 81$

b) $\log_{\frac{1}{2}} \frac{1}{8} = 3$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

c) $\log_3 1 = 0$

$3^0 = 1$

d) $\log_6 \frac{1}{36} = -2$

$6^{-2} = \frac{1}{36}$

2. Write the following in logarithmic form.

a) $4^2 = 16$

$\log_4 16 = 2$

b) $x^{2y} = z + 1$

$\log_x (z+1) = 2y$

c) $\left(\frac{1}{2}\right)^{-5} = 32$

$\log_{\frac{1}{2}} 32 = -5$

d) $\sqrt{x} = y$

$x^{1/2} = y$

$\rightarrow \log_x y = \frac{1}{2}$

3. Evaluate the given logarithms.

a) $\log_2 8$ $= 3$

b) $\log_{16} 4$ $= \frac{1}{2}$

c) $\log_5 \frac{1}{5}$ $= -1$

d) $\log_3 1$ $= 0$

e) $7^{\log_3 3^{2x}} - \log_{2x}\left((2x)^{7^{2x}}\right)$

$= 7^{2x} - 7^{2x}$

$= 0$

f) $16^{\log_2 2} - \log_x x^2$

$= 16^1 - 2$

$= 14$

4. Given that $\log_5 z = 3$ and $\log_5 y = 2$, evaluate each expression.

a) $\log_5(y^2 z)$

$\log_5(y^2 z) = \log_5 y^2 + \log_5 z$

$= 2\log_5 y + \log_5 z$

$= 2(2) + 3 = 7$

b) $\log_5 \sqrt[3]{\frac{z}{y}}$

$\log_5 \sqrt[3]{\frac{z}{y}} = \log_5 \left(\frac{z}{y}\right)^{1/3} = \frac{1}{3}\log_5 \frac{z}{y}$

$= \frac{1}{3}(\log_5 z - \log_5 y) = \frac{1}{3}(3-2) = \frac{1}{3}$

c) $\log_5(125y^7)$

$\log_5(125y^7) = \log_5 125 + \log_5 y^7$

$= 3 + 7\log_5 y$

$= 3 + 7(2) = 17$

d) $\dfrac{\log_5(125y^7)}{\log_5 25} = \dfrac{\log_5(125) + \log_5 y^7}{2}$

$= \dfrac{3 + 7\log_5 y}{2} = \dfrac{3+7(2)}{2}$

$= 17/2$

e) $\log_5(25y^2)\log_5(z)$

$= (\log_5(25) + \log_5 y^2)(3)$

$= (2 + 2\log_5 y)(3) = (2+2(2))3 = 18$

f) $\log_5(25y^2z)$

$= \log_5(25) + \log_5 y^2 + \log_5 z$

$= 2 + 2\log_5 y + 3$

$= 2 + 2(2) + 3 = 9$

5. Write each logarithm in expanded form.

a) $\log \sqrt[4]{xy} = \log(xy)^{1/4}$

$= \frac{1}{4}\log xy$

$= \frac{1}{4}(\log x + \log y)$

b) $\log \dfrac{xy}{z}$

$= \log xy - \log z$

$= \log x + \log y - \log z$

c) $\log \dfrac{\sqrt{x}\,\sqrt[3]{y}}{z^4} = \log \sqrt{x}\sqrt[3]{y} - \log z^4$

$= \log x^{1/2} + \log y^{1/3} - 4\log z$

$= \frac{1}{2}\log x + \frac{1}{3}\log y - 4\log z$

d) $\log x\sqrt{z} = \log x + \log z^{1/2}$

$= \log x + \frac{1}{2}\log z$

e) $\log x\sqrt{\dfrac{\sqrt{x}}{z}} = \log x + \log\sqrt{\dfrac{\sqrt{x}}{z}}$

$= \log x + \log\left(\dfrac{\sqrt{x}}{z}\right)^{1/2} = \log x + \frac{1}{2}\log\dfrac{x^{1/2}}{z}$

$= \log x + \frac{1}{2}(\log x^{1/2} - \log z) = \log x + \frac{1}{4}\log x - \frac{1}{2}\log z$

f) $\log \dfrac{\sqrt[3]{x^2+x+1}}{\sqrt[5]{y}} = \log\sqrt[3]{x^2+x+1} - \log\sqrt[5]{y}$

$= \log(x^2+x+1)^{1/3} - \log y^{1/5}$

$= \frac{1}{3}\log(x^2+x+1) - \frac{1}{5}\log y$

6. Write each logarithm in condensed form.

a) $\log_2 x + \log_2 7$

$= \log_2(7x)$

b) $\dfrac{1}{2}(\log x - \log y + \log z)$

$= \frac{1}{2}\log x - \frac{1}{2}\log y + \frac{1}{2}\log z$

$= \log x^{1/2} - \log y^{1/2} + \log z^{1/2}$

$= \log\dfrac{\sqrt{x}}{\sqrt{y}} + \log\sqrt{z} = \log\dfrac{\sqrt{x}\sqrt{z}}{\sqrt{y}} = \log\sqrt{\dfrac{xz}{y}}$

c) $\dfrac{1}{5}(\log_2 z + 2\log_2 y)$

$= \frac{1}{5}\log_2 z + \frac{2}{5}\log_2 y$

$= \log_2 z^{1/5} + \log_2 y^{2/5}$

$= \log_2\sqrt[5]{z} + \log_2\sqrt[5]{y^2} = \log_2\left(\sqrt[5]{z}\,\sqrt[5]{y^2}\right)$

$= \log_2\sqrt[5]{zy^2}$

d) $\dfrac{1}{3}(\log x - 2\log y + 3\log z)$

$= \frac{1}{3}\log x - \frac{2}{3}\log y + \log z$

$= \log\sqrt[3]{x} - \log\sqrt[3]{y^2} + \log z$

$= \log\left(\dfrac{\sqrt[3]{x}\cdot z}{\sqrt[3]{y^2}}\right)$

e) $2\ln x - \frac{1}{2}\ln(x^2+1)$

$= \ln x^2 - \ln(x^2+1)^{1/2}$

$= \ln x^2 - \ln\sqrt{x^2+1} = \ln\left(\frac{x^2}{\sqrt{x^2+1}}\right)$

f) $2\ln x + \frac{1}{2}\ln(x^2-1) - \frac{1}{2}\ln(x^2+1)$

$= \ln x^2 + \ln\sqrt{x^2-1} - \ln\sqrt{x^2+1}$

$= \ln x^2\sqrt{x^2-1} - \ln\sqrt{x^2+1}$

$= \ln\left(\frac{x^2\sqrt{x^2-1}}{\sqrt{x^2+1}}\right) = \ln\left(x^2\sqrt{\frac{x^2-1}{x^2+1}}\right)$

7. Solve the following logarithmic equations.

a) $\ln x = 3$

$e^3 = x$

b) $\log x + \log(x-1) = \log(4x)$

$\log(x(x-1)) = \log(4x)$

$\log(x^2-x) = \log 4x$

$x^2 - x = 4x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$\cancel{x=0}$ $x = 5$

since 0 is not in the domain of the original equation.

c) $\log_3(x+25) - \log_3(x-1) = 3$

$\log_3\left(\frac{x+25}{x-1}\right) = 3$

$3^3 = \frac{x+25}{x-1}$

$27(x-1) = x+25$

$27x - 27 = x + 25$

$26x = 52$

$x = 2$

d) $\log x + \log(x-3) = 1$

$\log(x(x-3)) = 1$

$10^1 = x^2 - 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ $\cancel{x=-2}$

since 0 is not in the domain of the original equation.

e) $\log_2(x-2) + \log_2(x+1) = 2$

$\log_2((x-2)(x+1)) = 2$

$2^2 = x^2 - x - 2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ $\cancel{x=-2}$

f) $\log_9(x-5) + \log_9(x+3) = 1$

$\log_9((x-5)(x+3)) = 1$

$9^1 = x^2 - 2x - 15$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x = 6$, $\cancel{x=-4}$

8. Solve the following exponential equations.

a) $3^x - 2 = 12$

$3^x = 14$

$\ln(3^x) = \ln(14)$

$x\ln 3 = \ln(14)$

$x = \frac{\ln(14)}{\ln(3)}$

b) $3^{1-x} = 2$

$\ln(3^{1-x}) = \ln 2$

$(1-x)\ln 3 = \ln 2$

$\ln 3 - x\ln 3 = \ln 2$

$-x\ln 3 = \ln 2 - \ln 3$

$x = \frac{\ln 2 - \ln 3}{-\ln 3}$

c) $4^x = 8^{x+1}$

$(2^2)^x = (2^3)^{x+1}$

$2^{2x} = 2^{3(x+1)}$

$2^{2x} = 2^{3x+3}$

$\log_2(2^{2x}) = \log_2(2^{3x+3})$

$2x = 3x+3$

$-3 = x$

d) $\frac{10}{1+e^{-x}} = 2$

$2 + 2e^{-x} = 10$

$2e^{-x} = 8$

$e^{-x} = 4$

$\ln(e^{-x}) = \ln 4$

$-x\ln e = \ln 4$

$-x(\log_e e) = \ln 4$

$-x(1) = \ln 4$

$x = -\ln 4$

e) $5^{2x+1} = 3^{x-1}$

$\ln(5^{2x+1}) = \ln(3^{x-1})$

$(2x+1)\ln 5 = (x-1)\ln 3$

$2x(\ln 5) + \ln 5 = x\ln 3 - \ln 3$

$2x(\ln 5) - x(\ln 3) = -\ln 5 - \ln 3$

$x(2\ln 5 - \ln 3) = -\ln 5 - \ln 3$

$x = \frac{-\ln 5 - \ln 3}{2\ln 5 - \ln 3}$

f) $e^{x+1} = 3$

$\ln(e^{x+1}) = \ln 3$

$(x+1)(\ln e) = \ln 3$

$(x+1)(1) = \ln 3$

$x = \ln 3 - 1$