College Algebra
Review of Fractions
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1 Multiplication

To multiply fractions, we simple multiply across the top, and multiply across the bottom:

$$\text{(a) } \frac{2}{3}\cdot\frac{7}{4}=\frac{14}{12}, \qquad \text{(b) } \frac{1}{2}\cdot -\frac{5}{3}=-\frac{5}{6}, \qquad \text{(c) } \frac{5}{9}\cdot\frac{6}{7}=\frac{30}{63}.$$

Notice that the (a) and (c) can be simplified:

$$\frac{14}{12}=\frac{7}{6}, \qquad \frac{30}{63}=\frac{10}{21}.$$

Another way to compute examples (a) and (c) would have been to first "cancel out" before multiplying. Since the first fraction of (a) has a 2 in the numerator and the second fraction has a 4 in the denominator, we may divide each of these numbers by their common factor 2, first. So the 2 becomes a 1, and the 4 becomes a 2.

$$\text{(a)}\frac{1\,\cancel{2}}{3}\cdot\frac{7}{2\,\cancel{4}}=\frac{1}{3}\cdot\frac{7}{2}=\frac{7}{6}.$$

Similarly, since 3 divides both 6 and 9, we can cancel out a 3 in (c):

$$\text{(c)}\frac{5}{3\,\cancel{9}}\cdot\frac{2\,\cancel{6}}{7}=\frac{5}{3}\cdot\frac{2}{7}=\frac{10}{21}.$$

CAREFUL! We can only "cancel out" if one of the numbers is in the numerator and the other is in the denominator. In other words **THE FOLLOWING IS INCORRECT:**

$$\frac{2}{3}\cdot\frac{4}{7}=\frac{1\,\cancel{2}}{3}\cdot\frac{2\,\cancel{4}}{7}=\frac{2}{21}.$$

It is incorrect because we "canceled out" two numerators. We need one to be a numerator and one to be a denominator.

Canceling out first is *not required* and sometimes, it's not possible. But you can always just multiply across the top, multiply across the bottom, and simplify later. However it may helpful to first cancel out. For example,

$$\frac{21}{9}\cdot\frac{10}{3}=\frac{210}{18},$$

and now we still need to simplify. But, if we first notice that we can cancel out a 3, we get

$$\frac{7\,\cancel{21}}{9}\cdot\frac{10}{1\,\cancel{3}}=\frac{70}{9},$$

and we are done, since 9 does not go into 70.

Lastly, let's talk about multiplying fractions by whole numbers. The only thing that changes is to remember that, for example, the whole number 5 is the same as the fraction $\frac{5}{1}$. So

$$5 \cdot \frac{3}{11} = \frac{5}{1} \cdot \frac{3}{11} = \frac{15}{11}.$$

A quick to do this would be to just remember that we multiply the whole number by the numerator, and keep the denominator the same:

$$2 \cdot \frac{7}{9} = \frac{14}{9}, \qquad \frac{7}{2} \cdot 3 = \frac{21}{2}, \qquad \frac{17}{3} \cdot -2 = -\frac{34}{3}.$$

Exercises. (1) $\frac{10}{3} \cdot \frac{6}{2}$ (2) $\frac{2}{3} \cdot \frac{1}{5}$ (3) $7 \cdot \frac{1}{2}$ (4) $15 \cdot \frac{2}{5}$

2 Division

To divide fractions, we multiply the top fraction by the reciprocal of the bottom fraction. "Reciprocal" means we flip it:

$$\text{(a)}\ \frac{-\frac{2}{3}}{\frac{1}{5}} = -\frac{2}{3} \cdot \frac{5}{1} = -\frac{10}{3}, \qquad \text{(b)}\ \frac{\frac{4}{7}}{\frac{2}{9}} = \frac{4}{7} \cdot \frac{9}{2} = \frac{2\,\cancel{4}}{7} \cdot \frac{9}{1\,\cancel{2}} = \frac{18}{7},$$

$$\text{(c)}\ \frac{\frac{11}{6}}{\frac{3}{4}} = \frac{11}{6} \cdot \frac{4}{3} = \frac{11}{3\,\cancel{6}} \cdot \frac{2\,\cancel{4}}{3} = \frac{22}{9}.$$

Now for some examples with whole numbers. Again, we just use the fact that a whole number can be changed into a fraction by putting it over 1. In other words, $4 = \frac{4}{1}$, $-2 = -\frac{2}{1}$, and so on. So the reciprocal of the whole number 4 is $\frac{1}{4}$, the reciprocal of -5 is $\frac{1}{-5}$, and so on.

$$\frac{4}{\frac{5}{7}} = 4 \cdot \frac{7}{5} = \frac{28}{5}, \qquad \frac{\frac{3}{2}}{5} = \frac{3}{2} \cdot \frac{1}{5} = \frac{3}{10}, \qquad \frac{\frac{7}{3}}{-7} = \frac{7}{3} \cdot \frac{1}{-7} = \frac{1\,\cancel{7}}{3} \cdot \frac{1}{-1\,\cancel{-7}} = \frac{1}{-3} = -\frac{1}{3}.$$

Exercises. (1) $\frac{3}{\frac{7}{3}}$ (2) $\frac{\frac{1}{3}}{\frac{2}{5}}$ (3) $\frac{\frac{11}{4}}{\frac{8}{3}}$ (4) $\frac{\frac{2}{3}}{4}$

3 Addition and Subtraction

We'll talk about addition and subtraction together because they are basically the same. To add/subtract fractions, we must have a common denominator. We can add $\frac{2}{3}$ and $\frac{5}{3}$ since they have the same denominator:

$$\frac{2}{3} + \frac{5}{3} = \frac{7}{3}.$$

So we add the numerators and leave the common denominator the same.

When we want to add/subtract fractions that don't have the same denominator, we have to multiply each of the fractions by what I like to call a "fancy number 1". Here's what I mean: Suppose we want to subtract

$$\frac{8}{5} - \frac{6}{7}.$$

We multiply $\frac{8}{5}$ by $\frac{7}{7}$ and we multiply $\frac{6}{7}$ by $\frac{5}{5}$. Notice that $\frac{7}{7}$ and $\frac{5}{5}$ are both equal to 1. So we aren't actually changing the values of our fractions, since we are multiplying both of them by 1! So $\frac{7}{7}$ and $\frac{5}{5}$ are both just a fancy way to write the number 1. We have

$$\frac{8}{5} \cdot \frac{7}{7} - \frac{6}{7} \cdot \frac{5}{5} = \frac{56}{35} - \frac{30}{35} = \frac{26}{35}.$$

We pick our fancy 1's from the denominators of the fractions in the problem. Here's more examples:

$$\begin{aligned} \frac{3}{5} - \frac{9}{2} &= \frac{3}{5} \cdot \frac{2}{2} - \frac{9}{2} \cdot \frac{5}{5} \\ &= \frac{6}{10} - \frac{45}{10} \\ &= -\frac{39}{10}. \end{aligned}$$

$$\begin{aligned} \frac{4}{7} + \frac{4}{11} &= \frac{4}{7} \cdot \frac{11}{11} + \frac{4}{11} \cdot \frac{7}{7} \\ &= \frac{44}{77} + \frac{28}{77} \\ &= \frac{72}{77}. \end{aligned}$$

$$\begin{aligned} 3 - \frac{5}{2} &= \frac{3}{1} - \frac{5}{2} \\ &= \frac{3}{1} \cdot \frac{2}{2} - \frac{5}{2} \cdot \frac{1}{1} \\ &= \frac{6}{2} - \frac{5}{2} \\ &= \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} \frac{3}{7} + 5 &= \frac{3}{7} + \frac{5}{1} \\ &= \frac{3}{7} \cdot \frac{1}{1} + \frac{5}{1} \cdot \frac{7}{7} \\ &= \frac{3}{7} + \frac{35}{7} \\ &= \frac{38}{7}. \end{aligned}$$

Exercises. (1) $\frac{5}{9} + \frac{1}{7}$ (2) $\frac{7}{6} - 2$ (3) $3 - \frac{1}{4}$ (4) $\frac{3}{7} + \frac{10}{3}$