Exam 2B, Fall 2015 – Problem 6

6. We wish to store .048114 moles of gas in a container of volume approximately $2m^3$ and at a temperature of appromimately 200 degrees Kelvin. For this quantity of gas, the ideal gas law says that $P = 4T/V$. Where P is pressure in dynes/cm^2, T is temperature in degrees Kelvin, and V is volume in m^3. Note that if $V = 2$ and $T = 200$, the pressure is 400. Due to errors, the actual values are $V = 2.01$ and $T = 203$. Use the total differential to estimate the pressure.

Solution: First, recall that error= "actual" − "approximate". The "actual" value refers to what *we* measured. We are told plainly in this problem what our "actual" values are, but in a different problem, we may have to interpret what the "actual" values are. So, the "actual" values refer to what the measurement is *actually.* So for example, the time that *we* recorded using a stopwatch, or the temperature that *we* measured using a thermometer are actual values.

On the other hand, the "approximate" values are those numbers that are *approximately* the numbers we got for our measurements that are easier to work with. Even if we weren't given approximate values for this problem, it would have been easy to pick $V_0 = 2$ and $T_0 = 200$ since they are close to our actual values. For this problem, we have:

$$\text{Actual Values: } V = 2.01 \text{ and } T = 203,$$
$$\text{Approximate Values: } V_0 = 2, T_0 = 200, \text{ and } P_0 = 400.$$

Notice that we put T_0, V_0, and P_0 for the approximate values. The point $(T_0, V_0 P_0)$ is a point that we *know* is on the surface described by $P = 4T/V$. From this point on, we are just finding the equation of the tangent plane at the point (T_0, V_0) in (dT, dV, dP)-coordinates, and then plugging in our actual and approximate values to finish the problem.

$$dP = \frac{\partial P}{\partial T} dT + \frac{\partial P}{\partial V} dV$$
$$dP = \frac{4}{V} dT - \frac{4T}{V^2} dV$$

In the same way we have been plugging in a fixed point (x_0, y_0) into our partial derivatives when finding the equation of the tangent plane at (x_0, y_0), we plug in (T_0, V_0) into our partials. We've just change some letters, but it's the same thing:

$$dP = \frac{4}{2} dT - \frac{4(200)}{(2)^2} dV$$
$$dP = 2dT - 200dV$$

At this point, we have an equation of a tangent plane at the point (V_0, T_0) for the surface given by the ideal gas law in (dT, dV, dP)-coordinates. So, what is our estimated value for P? We finish the problem by plugging in our actual and approximate values:

$$dP = 2dT - 200dV$$
$$P - P_0 = 2(T - T_0) - 200(V - V_0)$$
$$P - 400 = 2(203 - 200) - 200(2.01 - 2)$$
$$P = 404.$$