Introduction to Calculus
Course Notes

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Chapter 1

Preliminaries

1.1 Exponents

Let x denote a *variable*. In other words, x is a place-holder for an arbitrary number (any number at all, like $-4; 1.0001; 4,567; -43.9786$; or $\frac{2}{5}$). Let n be any positive whole number (like 1,2,3...). The nth power of the number x is

$$x^n = \underbrace{x \cdot x \cdot x \cdots x}_{n \text{ times}}$$

Example 1.1.1.

(a) $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

(b) $(-3)^2$

(c) 5^3

<u>**Zero Exponents:**</u> Any number (except 0) raised to the zero power is equal to one. In other words, if x is any number (except 0), then $x^0 = 1$. When we raise 0 to the zero power, we get 0. In other words, $0^0 = 0$.

Example 1.1.2.

(a) $3^0 = 1$.

(b) $(3.14)^0$

(c) $(-1,000,000)^0$

Negative Exponents: If a is any number, and $-n$ is a negative whole number, then we have:

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n.$$

Example 1.1.3.

(a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

(b) $(-4)^{-2}$

(c) $\frac{1}{7^{-1}}$

(d) x^{-2}

(e) $\frac{1}{x^{-2}}$

(f) $\frac{3}{x^{-2}}$

Be Careful! -3^2 is *not* the same thing as $(-3)^2$. When we write -3^2, what we mean is "3 squared, then make it negative." So

$$-3^2 = -(3^2) = -9.$$

On the other hand, when we write $(-3)^2$, we mean "multiply -3 by itself." So

$$(-3)^2 = (-3)(-3) = 9.$$

Concept Check: Try the following problems on your own:

(a) 3^4

(b) $(-4.87)^0$

(c) 4^{-3}

(d) $(-4)^2$

(e) -4^2

(f) $\dfrac{1}{3^{-1}}$

(g) 8^{-2}

1.1.1 Rules for Exponents

The Product Rule for Exponents: Let a be any number, and let n and m be positive whole numbers. Then

$$a^n \cdot a^m = a^{n+m}.$$

Example 1.1.4.

(a) $3^2 \cdot 3 = 3^{2+1} = 3^3 = 27$.

(b) $x^2 \cdot x^2$

(c) $2x^3 \cdot x^4$

(d) $x^{-7} \cdot x^7$

(e) $(-4x)(3x^7)$

The Quotient Rule for Exponents: Let a be any number, and let n and m be positive whole numbers. Then

$$\frac{a^n}{a^m} = a^{n-m}.$$

Example 1.1.5.

(a) $\dfrac{5^{10}}{5^9} = 5^{10-9} = 5^1 = 5.$

(b) $\dfrac{x^3}{x}$

(c) $\dfrac{x^3}{x^{-4}}$

(d) $\dfrac{2x^{-4}}{3x^5}$

Power-to-a-Power Rule for Exponents: Let a be any number, and let n and m be positive whole numbers. Then

$$(a^n)^m = a^{n \cdot m}$$

Example 1.1.6.

(a) $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64.$

(b) $(x^3)^{-1}$

(c) $(5^2)^0$

(d) $(x^2)^2$

Power-of-a-Product Rule for Exponents: Let a and b be any numbers, and let n be a positive whole number. Then

$$(a \cdot b)^n = a^n \cdot b^n.$$

Be Careful! We are *not* saying here that $(a+b)^n = a^n + b^n$. **That is not true!** In other words, it is *not true* that $(3+x)^2 = 3^2 + x^2$. Instead, what the rule says is true is that

$$(3 \cdot x)^2 = 3^2 \cdot x^2 = 9x^2.$$

Notice the difference: We are allowed to "distribute" the exponent to each number inside the parenthesis if we are *multiplying* the numbers in the parentheses. But we are *not* allowed to do this if we are adding the numbers in the parentheses.

Example 1.1.7.

(a) $(7x)^2 = 7^2x^2 = 49x^2$.

(b) $(2x^2)^3$

(c) $(x^4y^2)^3$

(d) $(2x^4y^2)^3$

Power-to-a-Quotient Rule for Exponents: Let a and b be any numbers, and let n be a positive whole number. Then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Example 1.1.8.

(a) $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$.

(b) $\left(\frac{2}{3}\right)^{-3}$

(c) $\left(\frac{7}{x}\right)^2$

Concept Check: Try the following problems on your own:

(a) x^2x^5

(b) $(x^2)^5$

(c) $2^{-2} \cdot 2^{10}$

(d) $3x^4 \cdot x^{-1}$

(e) $\dfrac{x^{-2}}{x^3}$

(f) $\dfrac{3x^4}{2x^2}$

(g) $(x^{-1})^4$

(h) $(2^2)^2$

(i) $(x^{-11})^0$

(j) $(2x)^2$

(k) $(2xy^2)^3$

(l) $2(xy^2)^3$

(m) $\left(\frac{1}{4}\right)^2$

(n) $\left(\frac{x}{3}\right)^{-2}$

1.2 Polynomials

A *polynomial* in the variable x is an expression that looks like

$$ax^n + bx^m + cx^\ell + \cdots \text{(and so on)}$$

where $a, b, c, \ldots$ are any numbers and $n, m, \ell, \ldots$ are positive whole numbers.

The expressions ax^n, bx^m, and cx^ℓ are each called *terms* of the polynomial. The *coefficient* of the term ax^n is the number a. The *degree* of a polynomial is the highest power that appears on x.

Example 1.2.1. Each of the following are examples of polynomials.

(a) $3x^2 + 4x + 1$.
- Terms: $3x^2$, $4x$, and 1
- Coefficients: 3, 4, and 1.
- Degree: 2, since the highest power on x is 2.

(b) $14x^{100} - 5x^{12} + x^4 - 7x^3 + x + 15$.
- Terms:
- Coefficients:
- Degree:

(c) $x + 3$.
- Terms:
- Coefficients:
- Degree:

(d) $x^2 - 5x + 3$.
- Terms:
- Coefficients:
- Degree:

(e) $17 - x^2 + 4x$.
- Terms:
- Coefficients:
- Degree:

1.2.1 Operations on Polynomials

Addition of Polynomials: To add polynomials, we "combine like terms". What this means is that we add the coefficients of two terms if they have the same power on x.

Example 1.2.2.

(a) $(2x^2 + 3x + 1) + (3x^2 + 4x - 4) = 5x^2 + 7x - 3.$

(b) $(x^3 + x + 4) + (x^2 + 4x)$

(c) $2x^2 + 2x + 2 + 4x^2 - 4x + 4$

(d) $x^5 - 7x^2 + 4 + x^3 + 3x$

Subtraction of Polynomials: To subtract polynomials, we first change the sign of each term in the polynomial that we are subtracting, then combine like terms.

Example 1.2.3.

(a) $$\begin{aligned}(2x^2 + 3x + 1) - (3x^2 + 4x - 4) &= 2x^2 + 3x + 1 - 3x^2 - 4x + 4 \\ &= -x^2 - x + 5.\end{aligned}$$

(b) $(x^3 + x + 4) - (x^2 + 4x)$

(c) $2x^2 + 2x + 2 - (4x^2 - 4x + 4)$

(d) $x^5 - 7x^2 + 4 - (x^3 + 3x)$

Multiplication of Polynomials: To multiply polynomials, multiply *each* term of the first polynomial by *each* term in the second polynomial, then add all these products up.

Example 1.2.4.

(a) $(2x^2+3x+1)(3x^2+4x-3)$

$$\begin{aligned}
&= (2x^2)(3x^2)+(2x^2)(4x)+(2x^2)(3)\\
&\qquad +(3x)(3x^2)+(3x)(4x)+(3x)(3)\\
&\qquad\qquad +(1)(3x^2)+(1)(4x)+(1)(3)\\
&= 6x^4+8x^3+6x^2\\
&\qquad +9x^3+12x^2+9x\\
&\qquad\qquad +3x^2+4x+3\\
&= 6x^4+17x^3+21x^2+13x+3.
\end{aligned}$$

(b) $(x^3+x+4)(x^2+4x)$

(c) $(2x^2+x)(4x+4)$

(d) $(x^5-7x^2)(x^3+3x)$

(e) $(x+1)^2$

Concept Check: Try the following problems on your own:

(a) List the terms, coefficients, and the degree of the following polynomial: $14x^3 + 3x^2 + 2x + 5$

- Terms:
- Coefficients:
- Degree:

(b) Simplify: $4x^4 - 3x^2 + 7x + 5 - 5x - 4x^2 + 2$

(c) Subtract: $4x^3 - 5x + 1 - (3x^3 + 3x^2 - 7x - 3)$

(d) Multiply: $(5x^2 + 3)(x - 1)$

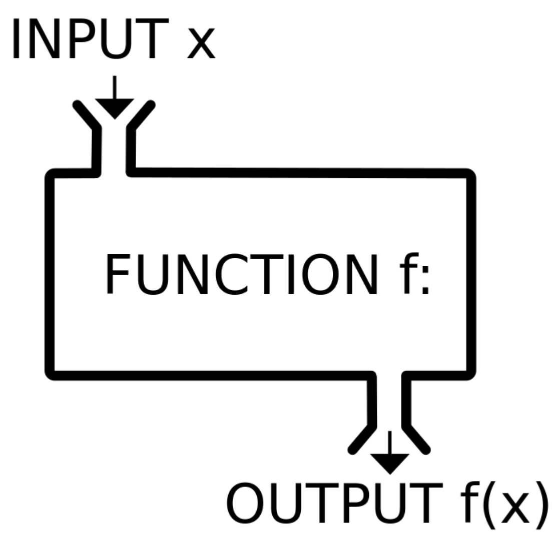
(e) Multiply: $(x + 1)(x - 2)$

1.3 Functions

A *function* is a machine that takes in a number as an input, and then outputs a number, according to a rule.

We usually like to name the machine/function f (we could also pick another letter, or even a word; its just a name of a machine). If x is a number that we are plugging in to the machine/function f, we write $f(x)$ to mean "plug x into the function f, and the output is $f(x)$".

The notation $f(x)$ does not mean f times x. Since f is the name of a machine/function, and not a number, it does not make sense to read $f(x)$ as "f times x," because we can only multiply numbers. Again we emphasize: The symbol $f(x)$ represents the output value we get when we plug the number x into the machine f. See the figure below:



If f is a function, what does it do with an input x? Well, each function comes with a rule that tells us what to do with an input. In this class, we will only consider *polynomial functions.*

1.3.1 Polynomial Functions

Polynomial functions are functions whose rule is given by a polynomial. In other words, the output of an input x looks like $f(x) = ax^n + bx^m + cx^\ell + \cdots$.

Example 1.3.1.

(a) The function f takes in an input x and outputs x^2. In other words, we write $f(x) = x^2$.

- $f(2) = 2^2 = 4$.
- $f(-5) = (-5)^2 = 25$.
- $f(☺) = ☺^2$.

(b) $f(x) = x + 5$.

- $f(2)$
- $f(-5)$
- $f(a)$
- $f(a + 1)$

(c) $g(x) = x^2 + 4x - 3$.

- $g(2)$
- $g(-5)$
- $g(a)$
- $g(a + b)$

Concept Check: Try the following problems on your own:

(a) $g(x) = x + 5$

- $g(2)$

- $g(-5)$
- $g(a)$
- $g(a+b)$

(b) $f(x) = x^2 + 2$

- $f(1)$
- $f(-1)$
- $f(4)$
- $f(-3)$

(c) $f(x) = (x+1)^3$

- $f(1)$
- $f(-1)$
- $f(☺)$
- $f(a+1)$

(d) $h(x) = 2x^2 + x^2 - 11$

- $h(1)$
- $h(2)$
- $h(3)$
- $h(c+5)$

1.3.2 Function Composition

If f and g are two functions, the *composition* of f and g is a new function which we call $f \circ g$, where the rule for $f \circ g$ is: Plug the input x into g first, then plug the output $g(x)$ into f. In other words

$$(f \circ g)(x) = f(g(x)).$$

Another way to think about it is to focus on the notation $f(g(x))$. What does this mean? Well, $f(☺)$ means we plug ☺ into the function f. So $f(g(x))$ means we plug in $g(x)$ into the machine f. But $g(x)$ means we plug x into g. So $f(g(x))$ means: plug x into g first, then plug the output $g(x)$ into f, like we already said above.

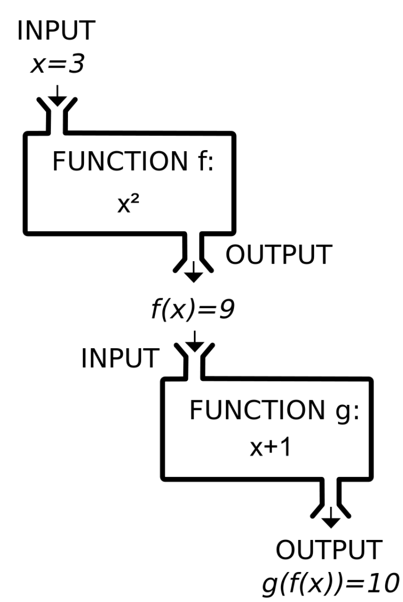
We can also do it the other way around:

$$(g \circ f)(x) = g(f(x)).$$

Example 1.3.2. Find both $(f \circ g)(x)$ and $(g \circ f)(x)$.

(a) $f(x) = x^2$ and $g(x) = x + 1$.

$$(f \circ g)(x) = f(g(x)) = (g(x))^2 = (x+1)^2.$$
$$(g \circ f)(x) = g(f(x)) = f(x) + 1 = (x^2) + 1 = x^2 + 1.$$



(b) $f(x) = 3x^2 + 5x + 2$ and $g(x) = x^2$.

(c) $f(x) = x^3$ and $g(x) = x + 1$.

Example 1.3.3. Use the same functions as in the previous example.

(a) Find $(f \circ g)(3)$ and $(g \circ f)(3)$.
Since $(f \circ g)(x) = (x + 1)^2$, we have

$$(f \circ g)(3) = (3 + 1)^2 = 4^2 = 16.$$

Since $(g \circ f)(x) = x^2 + 1$, we have

$$(g \circ f)(3) = 3^2 + 1 = 10.$$

(b) Find $(f \circ g)(1)$ and $(g \circ f)(1)$.

(c) Find $(f \circ g)(2)$ and $(g \circ f)(2)$.

Now, let's look at the function $h(x) = (x+1)^2 + 5$. Notice that this is really the composition of two other functions: This looks like we plugged $g(x) = x+1$ into $f(x) = x^2 + 5$.

$$f(g(x)) = (g(x))^2 + 5 = (x+1)^2 + 5.$$

In this example, we call f the "outside function" and we call g the "inside function", since we are plugging g into f.

Example 1.3.4. Identify the inside and outside functions.

(a) $h(x) = 2(x+4)^3$. This function looks like we are plugging in $g(x) = x+4$ into the function $f(x) = 2x^3$. So

- Inside: $g(x) = x + 4$.
- Outside: $f(x) = 2x^3$.

(b) $h(x) = 2(x^2 - 1)^2 + 3(x^2 - 1)$.

- Inside:
- Outside:

(c) $h(x) = (x+1)^5 - 7$.

- Inside:
- Outside:

(d) $h(x) = (x^2 + 3)^4$

- Inside:
- Outside:

Concept Check: Try the following problems on your own:

(a) Find both $(f \circ g)(x)$ and $(g \circ f)(x)$. Then find $(g \circ f)(1)$.

(i) $f(x) = x^3 + 4$ and $g(x) = 2x^2 + x + 1$.

- $(f \circ g)(x)$
- $(g \circ f)(x)$
- $(g \circ f)(1)$

(ii) $f(x) = x^2 + 1$ and $g(x) = x - 1$.

- $(f \circ g)(x)$
- $(g \circ f)(x)$
- $(g \circ f)(1)$

(iii) $f(x) = x^2 + 6x + 10$ and $g(x) = x + 5$.

- $(f \circ g)(x)$
- $(g \circ f)(x)$
- $(g \circ f)(1)$

(b) Identify the inside and outside functions.

(i) $(x + 1)^2 - 5$

- Inside:
- Outside:

(ii) $2(2x + 1)^2 + 7(2x + 1)$

- Inside:
- Outside:

(iii) $(x^2 + 1)^4$

- Inside:
- Outside:

Chapter 2

Derivative Rules for Polynomials

In this course we are focusing only on polynomial functions, or in other words, functions that look like $f(x) = 3x^2 - 2x + 4x^{10} + x - 6$. In a Calculus course, we would learn about even more functions. So as we learn the derivative rules, we will apply them only to polynomial functions for now. Later on in a Calculus course, these same rules can be easily applied to other functions.

So what *is* a derivative? We start with a function f. The *derivative* of the function f is a new function that comes from f. We call this new function f', pronounced "f prime". The way we obtain this new function f' is what the remainder of this course is about. Given f, how do we find its derivative, f'?

2.1 Basic Derivatives

2.1.1 Constant Rule

Let f be a function. And suppose that no matter what number you plug into the function f, it always outputs the number 7. In other words, $f(x) = 7$. This is an example of a *constant* function. We call it a constant function since the output is a constant number.

The first derivative rule we will learn is the derivative of a constant function:

The derivative of a constant function is zero.

So if we have a function $f(x) = 7$, then the derivative of f is 0. So we would write $f'(x) = 0$. The derivative function f' is a function whose output is always equal to zero.

2.1.2 The Power Rule

Next, we will consider a function f that looks like $f(x) = ax^n$, where a is any number and n is a positive whole number. So let's say $f(x) = 2x^5$. The Power Rule says

The derivative of ax^n is nax^{n-1}.

So for example, the derivative of the function $f(x) = 2x^5$ would be

$$f'(x) = 5 \cdot 2x^4 = 10x^4.$$

In other words, we multiply the power on x by the coefficient, and then also reduce the power on x by 1.

Example 2.1.1. Find the derivatives of the given functions using the constant rule and the power rule.

(a) $f(x) = 2x^3$

We multiply the power on x, which is 3, by the coefficient, which is 2. Then we reduce the power on x by one. So

$$f'(x) = 6x^2.$$

(b) $f(x) = 3x^4$

(c) $h(x) = x^2$

(d) $g(x) = 2x$

(e) $f(x) = 10x^{20}$

(f) $f(x) = -x^3$

Concept Check: Try the following problems on your own:
Find the derivatives of the given functions.

(a) $f(x) = 2x^2$

(b) $f(x) = x^7$

(c) $g(x) = -3x^4$

(d) $f(x) = 2$

(e) $h(x) = -3x^2$

(f) $f(x) = x$

(g) $f(x) = 3.78$

(h) $g(x) = 0$

(i) $g(x) = x^{300}$

(j) $g(x) = -2x$

2.1.3 Sums and Differences

Next we look at finding the derivative of a function that is a sum or difference. For example, what is the derivative of the function $f(x) = x^2 + 7x + 1$? The rule is:

The derivative of a sum (or difference) is the sum (or difference) of the derivatives of each term.

In other words, we take the derivative of each term, then add them up. So from our example above, the derivative of x^2 is $2x$, the derivative of $7x$ is 7, and the derivative of 1 is 0. So

$$f'(x) = 2x + 7 + 0 = 2x + 7.$$

Example 2.1.2. Find the derivatives of the given functions using the sum and difference rule.

(a) $f(x) = 2x^3 - 4x + 6$.

The derivative of $2x^3$ is $6x^2$; the derivative of $-4x$ is -4; the derivative of 6 is 0. So

$$f'(x) = 6x^2 - 4.$$

(b) $g(x) = 2x + 1$

(c) $f(x) = 2x^5 + 4x^3 - 2x$

(d) $g(x) = -x^3 - 2x + 5$

(e) $g(x) = x - 4$

Concept Check: Try the following problems on your own:
Find the derivatives of the given functions using the constant rule and the power rule.

(a) $f(x) = 2x^2 - 4x$

(b) $g(x) = 10x^2 + 10x - 10$

(c) $f(x) = 1 - x^2 - x$

(d) $f(x) = 5$

(e) $f(x) = x^5 + 5x^2 + 14x$

(f) $h(x) = x + 3$

(g) $h(x) = 4.5$

(h) $h(x) = x$

(i) $g(x) = 3x^4 + 5x - 4$

(j) $f(x) = x^3 + x^2 + x + 1$

2.2 The Product Rule

The product rule tells us how to take the derivative when we are multiplying two functions. The product rule says

$$\boxed{(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)g'(x).}$$

So the derivative of a product is "the derivative of the first times the second, plus the first times the derivative of the second."

Let's look at an example: Let $h(x) = (x+2)(x^2-5x)$. Notice that the function h is a product of two functions: $x+2$ is the first function, and x^2-5x is the second function. The derivative of $x+2$ is 1. The derivative of x^2-5x is $2x-5$. The product rule says to multiply the derivative of the first function by the second function, so: $1\cdot(x^2-5)$. Then we need to multiply the first function by the derivative of the second function, so: $(x+2)(2x-5)$. Then we add these up. Therefore

$$h'(x) = x^2-5+(x+2)(2x-5).$$

Now we need to simplify: $(x+2)(2x-5) = 2x^2-5x+4x-10 = 2x^2-x-10$. Hence

$$\begin{aligned} h'(x) &= x^2-5+(x+2)(2x-5) \\ &= x^2-5+2x^2-x-10 \\ &= 3x^2-x-15. \end{aligned}$$

Example 2.2.1. Find the derivatives of the given functions using the product rule.

(a) $f(x) = (2x^2+1)(x^3+x)$

Using the product rule, we get that the derivative of f is

$$\begin{aligned} f'(x) &= (4x)(x^3+x)+(2x^2+1)(3x^2+1) \\ &= 4x^4+4x^2+x^4+2x^2+3x^2+1 \\ &= 10x^4+9x^2+1 \end{aligned}$$

(b) $h(x) = (2x+1)(3x-3)$.

(c) $f(x) = (x^2 - 5)(4x)$.

(d) $g(x) = (2x^3 + 5x^2 + 3x - 1)(3x^2 + x - 2)$.

Concept Check: Try the following problems on your own:
Find the derivatives of the given functions.

(a) $f(x) = (-2x^2 + 3x)(4x)$

(b) $g(x) = (-2x^3 + 2x^2)(x^2 + 2x - 3)$

(c) $f(x) = (x^7)(x^2 + 3x + 4)$

(d) $f(x) = (x + 1)(2x - 5)$

(e) $h(x) = (3x + 2)(4x^2 - 5x)$

(f) $f(x) = (x^2)(3x^2 + 5x)$

(g) $f(x) = (-4x^3 + 7)(3x)$

(h) $g(x) = (x + 1)(x - 1)$

2.3 The Quotient Rule

The quotient rule tells us how to find the derivative when we divide one function by another function:

$$\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}.}$$

There is a common phrase that people use to remember this rule:

"Low dee high minus high dee low, all over low squared."

We refer to the bottom function as the "low" and the top function as the "high". And the word "dee" means derivative. So the rule is: the bottom function times the derivative of the top, minus the top times the derivative of the bottom, all divided by the bottom function squared.

Example 2.3.1. Find the derivatives of the given functions using the quotient rule.

(a) $h(x) = \dfrac{2x^2 + 4}{x + 2}$

$$\begin{aligned} h'(x) &= \frac{(x+2)(4x) - (2x^2+4)(1)}{(x+2)^2} \\ &= \frac{4x^2 + 8x - (2x^2 + 4)}{(x+2)^2} \\ &= \frac{2x^2 + 8x - 4}{(x+2)^2}. \end{aligned}$$

(b) $f(x) = \dfrac{4x + 1}{3x^2}$

(c) $\dfrac{x^2 + x - 1}{2x - 5}$

(d) $\dfrac{-3x^4 + 5x^3}{-3x + 7}$

(e) $\dfrac{x + 2}{x - 4}$

Concept Check: Try the following problems on your own:
Find the derivatives of the given functions.

(a) $f(x) = \dfrac{2x - 7x^2 + 1}{x^2}$

(b) $g(x) = \dfrac{7 - x^2 + x}{4x + 4}$

(c) $f(x) = \dfrac{x^3 + 2x^2 + 8x}{3x^2}$

(d) $h(x) = \dfrac{x + 2}{x}$

(e) $f(x) = \dfrac{1}{x}$

(f) $h(x) = \dfrac{x}{x^2}$

(g) $f(x) = \dfrac{x^5 + 4x}{5x + x^4}$

(h) $h(x) = \dfrac{1}{x^2}$

(i) $g(x) = \dfrac{3x - 1}{x^2}$

(j) $g(x) = \dfrac{1 - x}{3x^2 + 4x + 1}$

2.4 The Chain Rule

The chain rule tells us how to take the derivative when we have a composition of functions (see section 1.3), or in other words, when we have one function plugged into another function. For example, how do we find the derivative of $(x^2 + 1)^{50}$. Well, one way would be to multiply $x^2 + 1$ by itself 50 times, but obviously that would be too much work. This is one reason why the chain rule is important. Notice that $(x^2+1)^{50}$ is the function x^2+1 plugged into the function x^{50}. So $x^2 + 1$ is the inside function, and x^{50} is the outside function. Here is how the chain rule works:

Step 1: Identify the inside function. Now, we "forget" the inside function by treating it like a variable.

- In our example, we view x^2+1 as a variable. So just for now, treat it like you would treat x when taking derivatives.

Step 2: Take the derivative of the outside function, treating the inside function as a variable.

- Let's take the first part of the derivative of $(x^2 + 1)^{50}$, where we are treating $x^2 + 1$ like a variable. So in our minds, we are viewing the inside function as just some variable. What's the derivative of $(\text{variable})^{50}$? Well, that would be $50{\cdot}(\text{variable})^{49}$. But our "variable" is $x^2 + 1$. So for Step 2, the first part of the derivative of $(x^2+1)^{50}$ is
$$50(x^2 + 1)^{49}.$$
But this is not the end. We have one more thing to do in Step 3.

Step 3: Multiply what you got in Step 2 by the derivative of the inside function.

- In Step 2, we got $50(x^2+1)^{49}$. The derivative of the inside function x^2+1 is $2x$. Now we multiply these two things together to finish: The derivative of $(x^2+1)^{50}$ is

$$50(x^2+1)^{49} \cdot 2x = 100x(x^2+1)^{49}.$$

Example 2.4.1. Find the derivatives of the given functions using the chain rule.

(a) $f(x) = (3x+2)^4$.

The inside function is $3x+2$, take the derivative of $(3x+2)^4$ treating $3x+1$ like a variable. So we get $4(3x+2)^3$. Next we multiply this by the derivative of $3x+2$, which is 3. Hence our answer is

$$f'(x) = 4(3x+2)^3 \cdot 3 = 12(3x+2)^3.$$

(b) $g(x) = (4x^2+5x)^4$

(c) $g(x) = (x+1)^{100}$

(d) $f(x) = (2x^2+5x)^5 + (2x^2+5)^3$

(e) $f(x) = 3(x+2)^4 + 4(x+2)^3$

Concept Check: Try the following problems on your own:
Find the derivatives of the given functions.

(a) $f(x) = (4x^2 + 5x)^{10}$

(b) $f(x) = (2x+4)^8$

(c) $f(x) = (x^3 + 3x^2 + 6)^5$

(d) $f(x) = (x^2 + 4x + 7)^3$

(e) $f(x) = (3x + 5)^3 + (3x + 5)^2$

(f) $f(x) = (2x)^{10} + (2x)^4$

(g) $f(x) = 6(3x^2 + 4x + 2)^3 + 5(3x^2 + 4x + 2)^2$