Engineering Math II

Chapter 8 Activity 2 (Solutions)

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Exercises for §8.2.

Exercise 2. Integrate

$$\int_{\mathbb{D}} f da = \int_c^d \left(\int_a^b f(x,y)dx \right) dy = \int_a^b \left(\int_c^d f(x,y)dy \right) dx$$

in both orders for the following functions and rectangles.

d) $f(x,y) = e^{x-2y}$ where $\mathbb{D} = \{(x,y) : 0 \le x \le \log(3), 0 \le y \le \log(4)\}$.

e) $f(x,y) = \sin(x)\cos(y)$ where $\mathbb{D} = \{(x,y) : 0 \le x \le \pi, -\pi/2 \le y \le \pi/2\}$.

Solution:

d)

$$\begin{aligned}
\int_0^{\log(3)} \left(\int_0^{\log(4)} e^{x-2y} dy \right) dx &= \int_0^{\log(3)} -\frac{1}{2}\left(e^{x-2y}\right)\Big|_{y=0}^{y=\log(4)} dx \\
&= -\frac{1}{2}\int_0^{\log(3)} \left(e^{x-2\log(4)} - e^x\right) dx \\
&= -\frac{1}{2}\left(e^{x-2\log(4)} - e^x\right)\Big|_{x=0}^{x=\log(3)} \\
&= -\frac{1}{2}\left[\left(e^{\log(3)-2\log(4)} - e^{\log(3)}\right) - \left(e^{-2\log(4)} - e^0\right)\right] \\
&= -\frac{1}{2}\left[\left(e^{\log(3)}e^{-2\log(4)} - e^{\log(3)}\right) - \left(e^{-2\log(4)} - 1\right)\right] \\
&= -\frac{1}{2}\left[\left(e^{\log(3)}e^{\log(\frac{1}{16})} - e^{\log(3)}\right) - \left(e^{\log(\frac{1}{16})} - 1\right)\right] \\
&= -\frac{1}{2}\left[\left(\frac{3}{16} - 3\right) - \left(\frac{1}{16} - 1\right)\right] \\
&= \frac{15}{16}.
\end{aligned}$$

$$\begin{aligned}
\int_0^{\log(4)} \left(\int_0^{\log(3)} e^{x-2y} dx \right) dy &= \int_0^{\log(4)} \left(e^{x-2y}\right)\Big|_{x=0}^{x=\log(3)} dy \\
&= \int_0^{\log(4)} \left(e^{\log(3)-2y} - e^{-2y}\right) dy \\
&= \left(-\frac{1}{2}e^{\log(3)-2y} - \left(-\frac{1}{2}e^{-2y}\right)\right)\Big|_{y=0}^{y=\log(4)} \\
&= -\frac{1}{2}\left(e^{\log(3)-2y} - e^{-2y}\right)\Big|_{y=0}^{y=\log(4)} \\
&= -\frac{1}{2}\left[\left(e^{\log(3)-2\log(4)} - e^{-2\log(4)}\right) - \left(e^{\log(3)} - e^0\right)\right] \\
&= -\frac{1}{2}\left[\left(e^{\log(3)}e^{-2\log(4)} - e^{-2\log(4)}\right) - \left(e^{\log(3)} - e^0\right)\right] \\
&= -\frac{1}{2}\left[\left(e^{\log(3)}e^{-2\log(4)} - e^{-2\log(4)}\right) - \left(e^{\log(3)} - e^0\right)\right] \\
&= -\frac{1}{2}\left[\left(e^{\log(3)}e^{\log(\frac{1}{16})} - e^{\log(\frac{1}{16})}\right) - \left(e^{\log(3)} - 1\right)\right] \\
&= -\frac{1}{2}\left[\left(\frac{3}{16} - \frac{1}{16}\right) - (3-1)\right] \\
&= \frac{15}{16}.
\end{aligned}$$

e)

$$\begin{aligned}
\int_0^\pi \left(\int_{-\pi/2}^{\pi/2} \sin(x)\cos(y)dy \right) dx &= \int_0^\pi \sin(x)\ \left(\sin(y)\right)\big|_{y=-\pi/2}^{y=\pi/2} dx \\
&= \int_0^\pi \sin(x) \left((1-(-1)) \right) dx \\
&= 2 \int_0^\pi \sin(x)dx \\
&= 2\ \left(-\cos(x)\right)\big|_{x=0}^{x=\pi} \\
&= 2\left(-(-1)-(-1)\right) \\
&= 4. \\
\int_{-\pi/2}^{\pi/2} \left(\int_0^\pi \sin(x)\cos(y)dx \right) dy &= \int_{-\pi/2}^{\pi/2} \cos(y)\ \left(-\cos(x)\right)\big|_{x=0}^{x=\pi} dy \\
&= 2 \int_{-\pi/2}^{\pi/2} \cos(y)dy \\
&= 2\ \left(\sin(y)\right)\big|_{y=-\pi/2}^{y=\pi/2} \\
&= 2\left(1-(-1)\right) \\
&= 4.
\end{aligned}$$