P.3 - Polynomials

Def: • Let x be a variable (i.e. a "place-holder" for a number). A **monomial** is an expression of the form ax^n, where a is a real number, and n is a non-negative integer.

Ex: $3x^4$, $5x$, 3 ($= 3x^0$), πx, $-7x^{265}$

- The number "a" is called the **coefficient** of the monomial ax^n.

  Ex: From above: $3, 5, 3, \pi, -7$ are the coefficients.

- The power on x in the monomial ax^n is called the **degree** of the monomial.

  Ex: From above: $4, 1, 0, 1, 265$

- A **polynomial** is a sum of monomials

  Ex: $3x^4 + 5x + 3 + \pi x - 7x^{265}$

- Each monomial in a polynomial is called a **term** of the polynomial.

  Ex: Given $3x^2 + 2x + 1$, $3x^2$, $2x$, 1 are the terms.

Note: We usually write polynomials with descending degrees.

Adding / Subtracting Polynomials:

- "Combine like terms":

Ex: ① $(3x^2+5x-9)+(4x+1)$

$= 3x^2+9x-8$

$(3x^2+5x-9)-(4x+1)$

$=(3x^2+5x-9)+(-(4x+1))$

$=(3x^2+5x-9)+((-1)(4x+1))$

$=3x^2+5x-9+-4x-1$

$=3x^2+x-10$

② $(2x^2+2x+2)-(4x^2-4x+4)$

$=2x^2+2x+2-4x^2+4x-4$

$=-2x^2+6x-2$

Multiplying Polynomials

- Two ways to think about it:

① Remember the distributive property:

$$a(b+c)=ab+ac$$

$$(b+c)a=ab+ac$$

$(4x^2+2x+1)(3x+5) = (4x^2+2x+1)(3x) + (4x^2+2x+1)(5)$

Treat like the "a" above.

$= (4x^2)(3x) + (2x)(3x) + (1)(3x) + (4x^2)(5) + (2x)(5) + (1)(5)$

$= 12x^3 + 6x^2 + 3x + 20x^2 + 10x + 5$

$= 12x^3 + 26x^2 + 13x + 5$

② Multiply each term in the first polynomial by each term in the second.

Ex: $(-2x^2+3x+1)(x^2+2x-4)$

$= (-2x^2)(x^2) + (-2x^2)(2x) + (-2x^2)(-4) + (3x)(x^2) + (3x)(2x) + (3x)(-4) + (1)(x^2) + (1)(2x) + (1)(-4)$

$= -2x^4 - 4x^3 + 8x^2 + 3x^3 + 6x^2 - 12x + x^2 + 2x - 4$

$= -2x^4 - x^3 + 15x^2 - 10x - 4$

"FOIL": Ex: $(3x+2)(x-4)$

First + Outter + Inner + Last

$= (3x)(x) + (3x)(-4) + (2)(x) + (2)(-4)$

$= 3x^2 - 12x + 2x - 8$

$= 3x^2 - 10x - 8$

Squaring a binomial: Let A, B be monomials.

$$(A+B)^2 = (A+B)(A+B)$$
$$= AA + AB + BA + BB$$
$$= A^2 + AB + AB + B^2$$
$$= A^2 + 2AB + B^2$$

Ex $(x+3)^2 = x^2 + 2(x)(3) + 9$
$= x^2 + 6x + 9$

$$(x+3)^2 = (x+3)(x+3) = x \cdot x + x(3) + 3x + 3 \cdot 3$$
$$= x^2 + 3x + 3x + 9$$
$$= x^2 + 6x + 9$$

Difference of Squares:

$$(A+B)(A-B) = A^2 - B^2$$

Ex: $(x+3)(x-3) = x^2 - 9$

$$(x+3)(x-3) = (x)(x) + (x)(-3) + (3)(x) + (3)(-3)$$
$$= x^2 - 3x + 3x - 9$$
$$= x^2 - 9$$