2.8 - Combining Functions; Composite Functions.

- Let f be a function, given by the rule
$$f(x) = x^2 + 1.$$
Let g be a function, given by the rule
$$g(x) = \sqrt{x}.$$

Then:

$$(f+g)(x) = f(x) + g(x) = x^2+1 + \sqrt{x}$$
$$(f-g)(x) = f(x) - g(x) = x^2+1 - \sqrt{x}$$
$$(fg)(x) = f(x) \cdot g(x) = (x^2+1)(\sqrt{x})$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+1}{\sqrt{x}}.$$

Domains:

$$(f+g)(x) = x^2+1+\sqrt{x}$$
$$x \geq 0 \Rightarrow [0, \infty).$$
$$(f-g)(x) = x^2+1-\sqrt{x}$$
$$x \geq 0 \Rightarrow [0, \infty)$$
$$\begin{aligned}(fg)(x) &= (x^2+1)(\sqrt{x}) \\ &= (x^2+1)(x^{1/2}) \\ &= x^2 \cdot x^{1/2} + x^{1/2} \\ &= x^{5/2} + x^{1/2} \\ &= (\sqrt{x})^5 + \sqrt{x}\end{aligned}$$
$$x \geq 0 \Rightarrow [0, \infty)$$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2+1}{\sqrt{x}}$$

① $x \geq 0$, $[0, \infty)$

② $\sqrt{x} \neq 0$, so $x \neq 0$.

$\Longrightarrow \quad (0, \infty)$

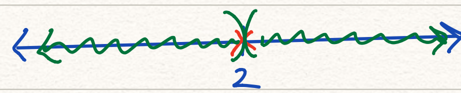
- Let $f(x) = x^2 - 6x + 8$

  $g(x) = x - 2$

$$\left(\frac{f}{g}\right)(x) = \frac{x^2-6x+8}{x-2} = \frac{(x-2)(x-4)}{x-2}$$

$$= x - 4 \quad , \quad x \neq 2.$$

<u>Domain</u> : $x \neq 2$



$(-\infty, 2) \cup (2, \infty)$

<u>Composite Functions</u>

- Plug one function into another

<u>Ex</u> : ① $f(x) = x^2 - 2$

$g(x) = x + 1$

$$(f \circ g)(x) = f(g(x)) = f(x+1)$$

$$= (x+1)^2 - 2$$

(2) $f(x) = x^2 + 1$

$g(x) = \sqrt{x}$

$$(f \circ g)(x) = \underbrace{f(g(x))}_{\text{"f of g of x"}} = f(\sqrt{x}) = (\sqrt{x})^2 + 1$$

$$= x + 1$$

Domain of $f \circ g$:

- Domain of $x+1$ is $(-\infty, \infty)$
- But the domain of $f \circ g$ is:

(The domain of $x+1$) $\cap$ (The domain of g)

$(-\infty, \infty) \cap [0, \infty)$

$[0, \infty)$

$$(g \circ f)(x) = g(f(x)) = \sqrt{x^2+1}$$

Domain of $g \circ f$:

(The domain of $\sqrt{x^2+1}$) $\cap$ (The domain of $f(x) = x^2+1$)

$(-\infty, \infty) \cap (-\infty, \infty)$

$(-\infty, \infty)$

$x^2 + 1 \geq 0$

$x^2 \geq -1$ $\rightarrow$ always true

③ $f(x) = x^2 - 4$

$g(x) = x^2 - 6x + 8.$

Find $f \circ g$ and $g \circ f$, and their domains.

$(f \circ g)(x) = (x^2 - 6x + 8)^2 - 4$

Domain : $(-\infty, \infty)$

$(g \circ f) = (x^2 - 4)^2 - 6(x^2 - 4) + 8$

Domain : $(-\infty, \infty)$

④ $f(x) = \dfrac{x^2 + 1}{x}$.

$g(x) = \sqrt{x}$

$$(f \circ g)(x) = f(g(x))$$

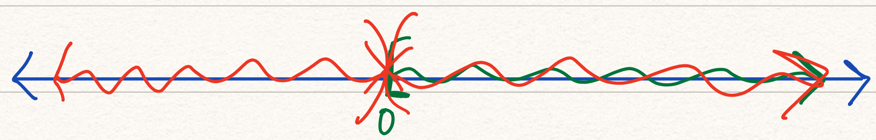
$$= \frac{(\sqrt{x})^2 + 1}{\sqrt{x}}$$

$$= \frac{x + 1}{\sqrt{x}}$$

$\sqrt{x} = 0$

$x = 0$

Domain : $\underbrace{[0, \infty)}_{\text{domain of } g} \cap \underbrace{\left((-\infty, 0) \cup (0, \infty) \right)}_{\text{domain of } \frac{x+1}{\sqrt{x}}}$



Domain : $(0, \infty)$