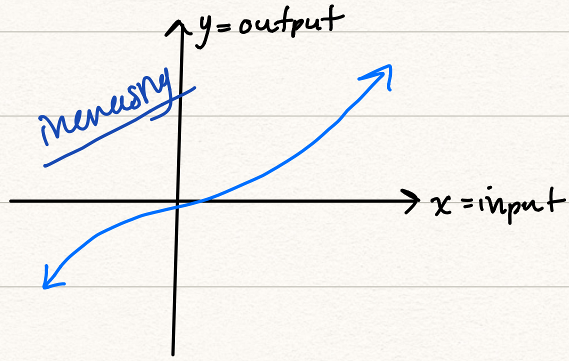
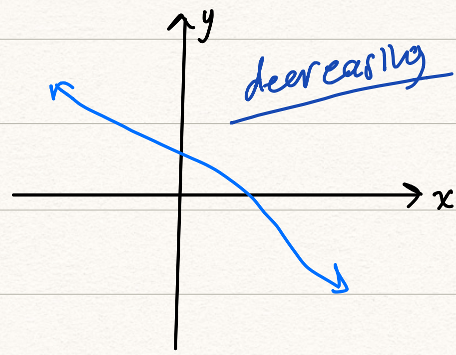
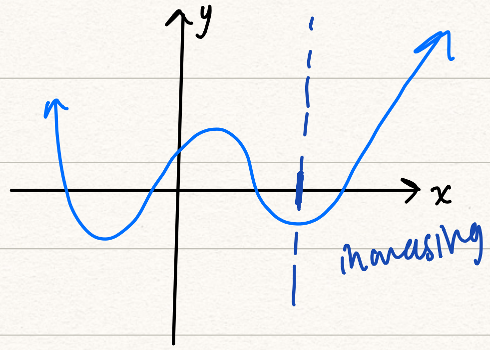
2.5 - Properties of Functions

Increasing/Decreasing: As the input values increase/decrease, so do the corresponding outputs

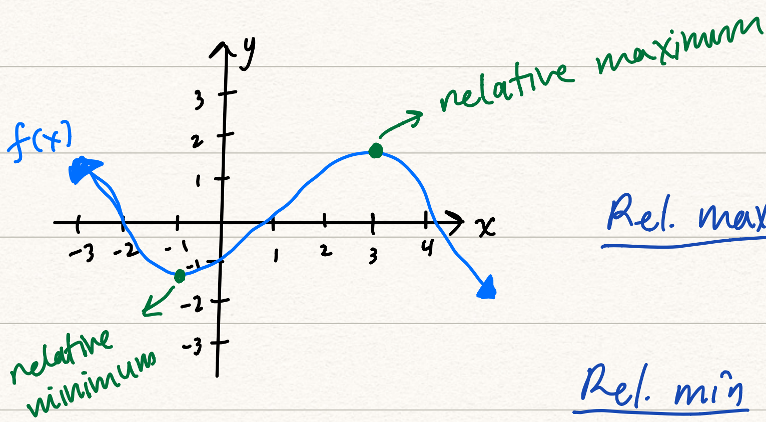






Relative Maximum/Minimum Values of a Function:

- An output value that is bigger/smaller than the output values around it.



Rel. max: at $x = 3$, w/ max value $f(3) = 1.5$

Rel. min: at $x = -1$, w/ min value $f(-1) = -1.2$

Even/Odd Functions:

- **Even function**: Symmetric about the y-axis.
  Check algebraically: $f(-x) = f(x)$
- **Odd function**: Symmetric about the origin
  Check algebraically: $f(-x) = -f(x)$

Ex: ① $f(x) = 2x^4 + 4$

Lets check: $f(-x) = 2(-x)^4 + 4$

$= 2x^4 + 4$

$= f(x)$

$\Rightarrow$ Even function!

② $g(x) = 5x^3 - 3x$

$g(-x) = 5(-x)^3 - 3(-x)$

$= (5)(-x^3) - (3)(-x)$

$= (5)(-1)(x^3) - (3)(-1)(x)$

$= -5x^3 + 3x$

$= -(5x^3 - 3x)$

$= -g(x)$

$\Rightarrow$ Odd function

③ $f(x) = \dfrac{1}{x^2+4}$

$f(-x) = \dfrac{1}{(-x)^2+4} = \dfrac{1}{x^2+4} = f(x)$

$\Rightarrow$ Even function.

④ $h(x) = \dfrac{x^2 - 2x}{5x^4 + 7}$

$$h(-x) = \frac{(-x)^2 - 2(-x)}{5(-x)^4 + 7}$$

$$= \frac{x^2 + 2x}{5x^4 + 7}$$

$$-h(x) = -\frac{(x^2 - 2x)}{5x^4 + 7} = \frac{-x^2 + 2x}{5x^4 + 7}$$

So $h(-x) \neq h(x)$

$h(-x) \neq -h(x)$

Hence h is neither even nor odd.

⑤ $f(x) = \dfrac{x^4 + 3}{2x^3 - 3x}$, $f(-x) = \dfrac{(-x)^4 + 3}{2(-x^3) - 3(-x)} = \dfrac{x^4 + 3}{-2x^3 + 3x}$

$$= \frac{x^4 + 3}{-(2x^3 - 3x)}$$

$$= -\frac{x^4 + 3}{2x^3 - 3x}$$

$= -f(x) \Rightarrow f$ is odd

⑥ $h(x) = \dfrac{x}{x^5 - 3x}$

$$h(-x) = \frac{-x}{(-x)^5 - 3(-x)}$$

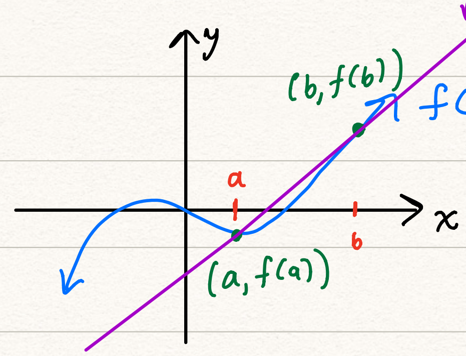
$$= \frac{-x}{-x^5 + 3x}$$

$$= \frac{(-1)x}{-(x^5 - 3x)} = \frac{x}{x^5 - 3x} = h(x) \Rightarrow h \text{ is even}$$

Average rate of change (of a function)

- from an input value $x=a$ to input value $x=b$ is the slope of the line between the points $(a, f(a))$ and $(b, f(b))$,

$$\boxed{\frac{f(b)-f(a)}{b-a}}$$



Ex: Avg. rate of change of

$f(x) = x^2+1$ $\qquad$ $f(3) = 3^2+1 = 10$

from $x=0$ to $x=3$.

$$\rightarrow \frac{f(3)-f(0)}{3-0} = \frac{10-1}{3} = 3.$$