2.4 - Functions

Def: A <u>function</u> is a machine that takes in real numbers as inputs, and spits out real numbers as outputs, such that each input <u>has exactly one output</u>.

according to a rule

Ex: Let f be a function defined by the rule x^2+1, where x denotes an arbitrary input.

$$2 \xrightarrow{\text{input}} \boxed{f} \xrightarrow{\text{output}} (2)^2+1 = 5$$

$$3 \longrightarrow \boxed{f} \longrightarrow (3)^2+1 = 10$$

Notation: The output value corresponding to an input x is denoted by $f(x)$, read "f of (x)".

* Not "f times x" *

$$2 \longrightarrow \boxed{f} \longrightarrow f(2) = 5$$

$$3 \longrightarrow \boxed{f} \longrightarrow f(3) = 10$$

$$\Delta \longrightarrow \boxed{f} \longrightarrow f(\Delta) = \Delta^2+1$$

$$a^2-7 \longrightarrow \boxed{f} \longrightarrow f(a^2-7) = (a^2-7)^2+1$$

- Sometimes the rule of a function is given by an eqn, like $y = x^2-6x+8$, where y = output value.

So, if g is a function defined by the rule

$$y = x^2-6x+8,$$

then $g(x) = x^2 - 6x + 8$.

Then: ① $g(3) = (3)^2 - 6(3) + 8$

$= 9 - 18 + 8$

$= -9 + 8$

$= -1$

② $g(\frac{1}{2}) = (\frac{1}{2})^2 - 6(\frac{1}{2}) + 8$

$= \frac{1}{4} - 3 + 8$

$= \frac{-11}{4} + \frac{32}{4}$

$= \frac{21}{4}$

③ $g(x+h) = (x+h)^2 - 6(x+h) + 8$

$$\cancel{g(x) = x^2 - 6x + 8}$$
$$\cancel{g(x+h) = x^2 - 6x + 8 + h}$$

Domain of a funtion

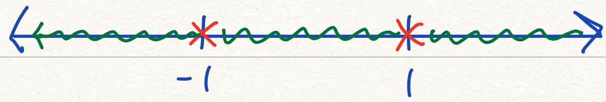
Def: The domain of a function is the set of all numbers that "we are allowed to plug in".

Note: We usually find the domain by finding numbers that are not allowed to be plugged in.

Ex ① $f(x) = \dfrac{1}{1-x^2}$

$$1 - x^2 = 0$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$
$$x = \pm 1$$

We can't plug in 1 or -1.



Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

② $g(x) = \sqrt{x+3}$

$$x + 3 \geq 0$$
$$x \geq -3$$

Domain: $[-3, \infty)$

③ $h(x) = \dfrac{1}{\sqrt{x-1}}$

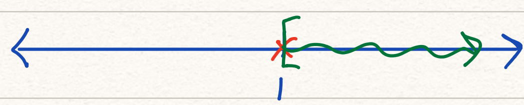
① Need denom. $\neq 0$ $\longrightarrow$ $\sqrt{x-1} = 0$

$x - 1 = 0$

$x = 1 \rightarrow$ so, x cannot be 1

② Need $x - 1 \geq 0$

$x \geq 1$ $\rightarrow$ so, our input has to be bigger or equal to 1



So, Domain: $(1, \infty)$

④ $P(x) = 3x^2 + 2x - 1$

Domain: $(-\infty, \infty)$

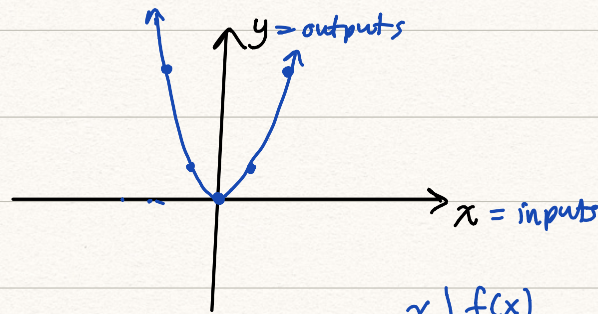
Range of a function:

Def: The range of a function is the set of all numbers that are possible outputs of the function.

Ex ① $f(x) = x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

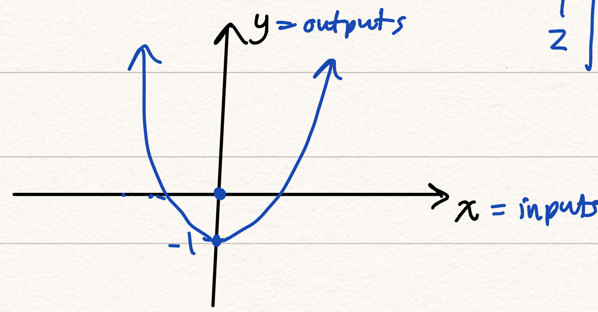


x	f(x)
-2	4
-1	1
0	0
1	1
2	4

② $f(x) = x^2 - 1$

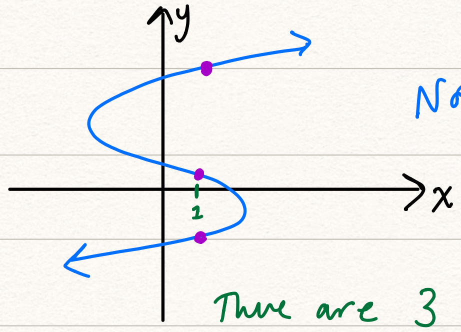
Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$



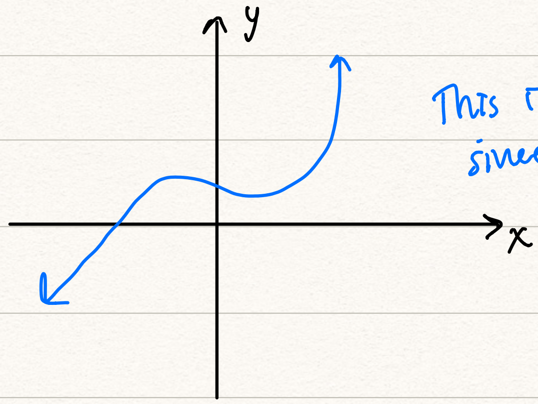
Remember: Functions assign exactly one output value for each input value.

Vertical Line test:



Not a function!

There are 3 output values for the input value $x = 1$.
So, <u>not</u> a function.



This is a function since the graph passes the vertical line test.