5.2 - Systems of Linear Equations in Three Variables.

Ex ①
$$\begin{cases} 3y - z = 5 \\ 2x + y + z = 9 \\ x + 2y + 2z = 3 \end{cases}$$

Step 1: Interchange eqns to get the first eqn with a coeff. of 1 on x.

$$\begin{cases} x + 2y + 2z = 3 \\ 2x + y + z = 9 \\ 3y - z = 5 \end{cases}$$

Step 2: Eliminate x terms from 2nd & 3rd eqn.

$$\begin{array}{l} 2x + y + z = 9 \\ +(-2)(x + 2y + 2z = 3) \\ \hline 0x - 3y - 3z = 3 \end{array}$$

add multiple of 1st eqn to 2nd eqn to get a new 2nd eqn.

So
$$\begin{cases} x + 2y + 2z = 3 \\ -3y - 3z = 3 \\ 3y - z = 5 \end{cases}$$

→ Now get a coeff. of 1 on y in 2nd eqn.

Mult. 2nd eqn by $-\frac{1}{3}$:

$$\left(-\tfrac{1}{3}\right)(-3y - 3z = 3) \rightarrow y + z = -1$$

$$\begin{cases} x + 2y + 2z = 3 \\ y + z = -1 \\ 3y - z = 5 \end{cases}$$

Step 3: Eliminate y from 3rd eqn.

$$\begin{array}{r} 3y - z = 5 \\ +(-3)(y + z = -1) \\ \hline 0y - 4z = 8 \end{array}$$

$$-4z = 8$$

$$\boxed{z = -2}$$

Step 4: Plug z into 2nd eqn to find y:

$$y + z = -1$$

$$y + (-2) = -1$$

$$\boxed{y = 1}$$

Step 5: Plug y & z into 1st eqn to find x:

$$x + 2y + 2z = 3$$

$$x + 2(1) + 2(-2) = 3$$

$$x - 2 = 3$$

$$\boxed{x = 5}$$