5.1 - Solving Systems of Linear Equations in Two Variables

A system of linear equations:

$$\begin{cases} 2x - y = 5 \\ x + 2y = 5 \end{cases}$$

Recall: $2x - 1 = 5$

→ Trying to find an x-value that makes the equation true.

Now: $\begin{cases} 3x + 4y = 1 \\ 7x - y = 2 \end{cases}$

→ Trying to find x-value(s) and y-value(s) that make both equations true.

- 3 possible solutions:
  1. Unique solution, i.e. exactly 1 x-value & 1 y-value satisfying the system. Eg.: $x = 4$, $y = \frac{1}{2}$
  2. Infinitely many solutions.
  3. No solution

The Substitution Method

Ex: ① $\begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases}$

From the second eqn. $y = 9 + 2x$ *

plug the y-value in to first eqn. $2x - 5(9 + 2x) = 3$

$2x - 45 - 10x = 3$

$-8x = 48$

$\boxed{x = -6}$

pick either eqn to plug $x=-6$ into: $y - 2(-6) = 9$

$\boxed{y = -3}$

② $\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases}$

solve for y in first eqn. (arbitrary choice) $4x + 2y = 12$

$2y = 12 - 4x$

$y = 6 - 2x$ *

plug $y = 6 - 2x$ into other eqn: $-2x - (6 - 2x) = -6$ *

$-2x - 6 + 2x = -6$

$-6 = -6$ → True statement!

So: $x = t$

$y = 6 - 2t$

③ $\begin{cases} x+y=3 \\ 2x+2y=9 \end{cases}$

$\rightarrow y = 3 - x$

$\rightarrow 2x + 2(3-x) = 9$

$2x + 6 - 2x = 9$

$6 = 9$ → Not true!

So, no solution!

Elimination Method

Ex: ① $\begin{cases} 2x+3y=21 \\ 3x-4y=23 \end{cases}$

(3) $(2x+3y=21)$

$6x + 9y = 63$

(-2) $(3x-4y=23)$

$-6x + 8y = -46$

$6x+9y = 63$

$-6x+8y = -46$

$0x + 17y = 17$

$17y = 17$

$\boxed{y = 1}$

$2x + 3(1) = 21$

$2x = 18$

$\boxed{x = 9}$

- If we get something like $6 = 6$ (any true sentence), then we have infinitely many solutions.

- If we get something like $-3 = 0$ (any false sentence), there are no solutions.