

5.1 - Solving Systems of Linear Equations in Two Variables

A system of linear equations:

$$\begin{cases} 2x - y = 5 \\ x + 2y = 5 \end{cases}$$

Recall: $2x - 1 = 5$

→ Trying to find an x -value that makes the equation true.

Now:
$$\begin{cases} 3x + 4y = 1 \\ 7x - y = 2 \end{cases}$$

→ Trying to find x -value(s) and y -value(s) that make both equations true.

• 3 possible solutions:

- ① Unique solution, i.e. exactly 1 x -value & 1 y -value satisfying the system. Eg.: $x = 4$, $y = \frac{1}{2}$
- ② Infinitely many solutions.
- ③ No solution

The Substitution Method

$$\text{Ex: ①} \begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases}$$

From the second eqn. $y = 9 + 2x$ *

Plug the y-value in to first eqn.

$$2x - 5(9 + 2x) = 3$$

$$2x - 45 - 10x = 3$$

$$-8x = 48$$

$$x = -6$$

Pick either eqn to plug $x = -6$ into:

$$y - 2(-6) = 9$$

$$y = -3$$

$$\text{②} \begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases}$$

Solve for y in first eqn.

(arbitrary choice)

$$4x + 2y = 12$$

$$2y = 12 - 4x$$

$$y = 6 - 2x$$
 *

plug $y = 6 - 2x$ into other eqn:

$$-2x - (6 - 2x) = -6$$
 *

$$-2x - 6 + 2x = -6$$

$$-6 = -6$$

→ True statement!

$$\text{So: } x = t$$
$$y = 6 - 2t$$

$$\textcircled{3} \begin{cases} x + y = 3 \\ 2x + 2y = 9 \end{cases}$$

$$\rightarrow y = 3 - x$$

$$\rightarrow 2x + 2(3 - x) = 9$$

$$2x + 6 - 2x = 9$$

$$6 = 9 \rightarrow \text{Not true!}$$

So, no solution!

Elimination Method

$$\text{Ex: } \textcircled{1} \begin{cases} 2x + 3y = 21 \\ 3x - 4y = 23 \end{cases}$$

$$\textcircled{3} \begin{cases} 2x + 3y = 21 \\ 6x + 9y = 63 \end{cases}$$

$$\textcircled{-2} \begin{cases} 3x - 4y = 23 \\ -6x + 8y = -46 \end{cases}$$

$$6x + 9y = 63$$

$$-6x + 8y = -46$$

$$0x + 17y = 17$$

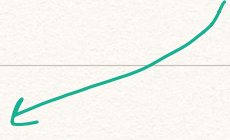
$$17y = 17$$

$$\boxed{y = 1}$$

$$2x + 3(1) = 21$$

$$2x = 18$$

$$\boxed{x = 9}$$



• If we get solutions. $\wedge 6 = 6$, then we have infinitely many solutions.
something like (any true sentence)

• If we get something like $-3 = 0$ (any false sentence), there are no solutions.