4.4 — Exponential & Logarithmic Equations

Ex: ① $25^x = 125$

$$(5^2)^x = 5^3$$

$$5^{2x} = 5^3$$

$$\log_5(5^{2x}) = \log_5(5^3)$$

$$2x = 3$$

$$x = \frac{3}{2}$$

② $9^x = 3^{x+1}$

$$(3^2)^x = 3^{x+1}$$

$$3^{2x} = 3^{x+1}$$

$$2x = x+1$$

$$x = 1$$

③ $5^{2x-3} = 3^{x+1}$

$$\ln(5^{2x-3}) = \ln(3^{x+1})$$

$$(2x-3)(\ln 5) = (x+1)(\ln 3)$$

$$2x(\ln 5) - 3(\ln 5) = x(\ln 3) + \ln 3$$

$$2x(\ln 5) - x(\ln 3) = 3(\ln 5) + \ln 3$$

$$\frac{x(2(\ln 5) - (\ln 3))}{(2(\ln 5) - (\ln 3))} = \frac{3(\ln 5) + \ln 3}{(2(\ln 5) - (\ln 3))}$$

$$x = \frac{3(\ln 5) + (\ln 3)}{2(\ln 5) - (\ln 3)}$$

④ $\log_2(x-3) + \log_2(x-4) = 1$

equals

$\log_{2}\big((x-3)(x-4)\big) = 1$

raised to the

$\log_3 9 = 2$

$3^2 = 9$

$2^1 = (x-3)(x-4)$

$2 = x^2 - 7x + 12$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$\rightarrow \boxed{x=5}, \quad \cancel{x=2}$

⑤ $\log_4 x + \log_4(x+1) = \log_4(x-1) + \log_4 6$

$\log_4\big(x(x+1)\big) = \log_4\big(6(x-1)\big)$

using the one-to-one property of logs

$x(x+1) = 6(x-1)$