4.3 - Rules of Logarithms

Let M, N be real numbers.

① Product Rule

$$\log_a MN = \log_a M + \log_a N$$

Ex ① $\log_3(27) = \log_3(9 \cdot 3) = \log_3 9 + \log_3 3 = 2 + 1 = 3$

② $\ln((x+1)(x+2)) = \ln(x+1) + \ln(x+2)$

③ $\log x + \log y = \log xy$

② Quotient Rule

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

Ex ① $\log_3\left(\frac{x+1}{3x^2-7}\right) = \log_3(x+1) - \log_3(3x^2-7)$

② $\log_3\left(\frac{9}{5}\right) = \log_3 9 - \log_3 5$

$= 2 - \log_3 5$

③ $\log\left(\frac{x}{4}\right) = \log x - \log 4$

③ Power Rule

$$\log_a M^r = r \cdot \log_a M$$

Ex ① $\ln 5^7 = 7 \cdot \ln 5$

② $\log 5^{3/2} = \frac{3}{2}\log 5$

③ $\log_3 \sqrt{x} = \log_3 x^{1/2} = \frac{1}{2} \cdot \log_3 x$

~~④ $\log(x^2+1) = 2\log(x+1)$~~ not correct

④ $\log(x^2+1)^3 = 3\log x^2+1$

⑤ $\log \frac{1}{\sqrt[3]{x}} = \log \frac{1}{x^{1/3}} = \log x^{-1/3} = -\frac{1}{3}\log x$

A Ton of Examples

- Given that $\log_5 z = 3$ and $\log_5 y = 2$, evaluate:

① $\log_5(yz) = \log_5 y + \log_5 z = 2 + 3 = 5$

② $\log_5(125y^7) = \log_5 125 + \log_5 y^7$
$= 3 + 7 \cdot \log_5 y$
$= 3 + 7 \cdot 2$
$= 17$

③ $\log_5 \sqrt{\frac{z}{y}} = \log_5 \left(\frac{z}{y}\right)^{1/2} = \frac{1}{2} \cdot \log_5 \frac{z}{y}$
$= \frac{1}{2} \cdot (\log_5 z - \log_5 y)$
$= \frac{1}{2} \cdot (3 - 2) = \frac{1}{2} \cdot$

④ $\log_5 \frac{y}{z} = -1$

⑤ $\log_5 y^2 z^3 = \log_5 y^2 + \log_5 z^3$
$= 2\log_5 y + 3\log_5 z$
$= 2 \cdot 2 + 3 \cdot 3$
$= 4 + 9$
$= 13$

- Write each expression in expanded form

① $\log_2 \frac{x^2(x-1)^3}{(2x+1)^4} = \log_2 x^2(x-1)^3 - \log_2 (2x+1)^4$

$= \log_2 x^2 + \log_2 (x-1)^3 - 4 \cdot \log_2 (2x+1)$

$= 2\log_2 x + 3\log_2 (x-1) - 4\log_2 (2x+1)$

② $\ln \sqrt{x^3 y^2 z^5} = \frac{1}{2}(\ln x^3 + \ln y^2 + \ln z^5)$

$= \frac{1}{2}(3\ln x + 2\ln y + 5\ln z)$

$= \frac{3}{2}\ln x + \ln y + \frac{5}{2}\ln z$

③ $\log \frac{2x-1}{x+4} = \log 2x-1 - \log x+4$

④ $\ln \sqrt{\frac{4xy}{z}} = \frac{1}{2}(\ln 4xy - \ln z)$

$= \frac{1}{2}(\ln 4 + \ln x + \ln y - \ln z)$