4.2 - Logarithmic Functions

Recall: • Section 2.9 - Inverse Functions

If $f(x) = x+2$, then $f^{-1}(x) = x-2$.

$$x \rightarrow \boxed{f} \rightarrow x+2 \rightarrow \boxed{f^{-1}} \rightarrow x$$

• Section 4.1 - Exponential Functions

$f(x) = 3^x$, $f(x) = \left(\frac{1}{3}\right)^x$

Now: Find inverse function for exponential function $f(x) = a^x$.

Step 1: $y = a^x$

Step 2: $x = a^y$

Step 3: Solve for y.

$\rightarrow$ Need logarithmic functions.

Def: A logarithmic function is a function f of the form

$f(x) = \log_b(x) =$ the number y so that $b^y = x$.

Ex: ① $\log_3(9)$ = the number y so that

$$3^y = 9$$

$= 2$

② $\log_2(16) = 4$

③ $\log_2\left(\frac{1}{4}\right) = 2$ (because $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$)

④ $\log_5(10)$ = some awful decimal.

- $\log_a(x)$ is the inverse function to a^x.

Check:

① $\log_a(a^x) = x$ ✓

② $a^{\log_a(x)}$ → the number we need to raise "a" to in order to get x

$= x$

* The domain of logarithmic functions is $(0, \infty)$.

$\log_3(0)$ = the number y so that $3^y = 0$

= none!

Ex: Write each exponential equation in logarithmic form

① $4^3 = 64$

$\rightarrow \log_4 64 = 3$

② $a^{-2} = 7$

$\rightarrow \log_a 7 = -2$

③ $2^{10} = 1024$

$\rightarrow \log_2 1024 = 10$

④ $2a^3 - 3 = 10$

$\rightarrow 2a^3 = 13$

$\rightarrow a^3 = \frac{13}{2}$

$\rightarrow \log_a \frac{13}{2} = 3$

Ex: Other way

① $\log_3 243 = 5$

$\rightarrow 3^5 = 243$

② $\log_a N = x$

$a^x = N$

③ $\log_2 64 = 6$

$\rightarrow 2^6 = 64$

④ $\log_v u = w$

$v^w = u$

Basic Properties of Logarithms

For any base $a > 0$, and $a \neq 1$:

1. $\log_a a = 1$
2. $\log_a 1 = 0$
3. $\log_a a^x = x$
4. $a^{\log_a x} = x$

Ex: ① $5^{\log_5 7} = 7$

② $2^{\log_2 7} + \log_5 5^{-3}$

$\rightarrow \ 7 + -3 = 4$

Recall: The exponential function $f(x) = a^x$ has domain $(-\infty, \infty)$, and range $(0, \infty)$

So: The logarithmic function $f(x) = \log_a x$ has domain $(0, \infty)$ and range $(-\infty, \infty)$.

Ex: Find the domain of $f(x) = \log_3 (2-x)$.

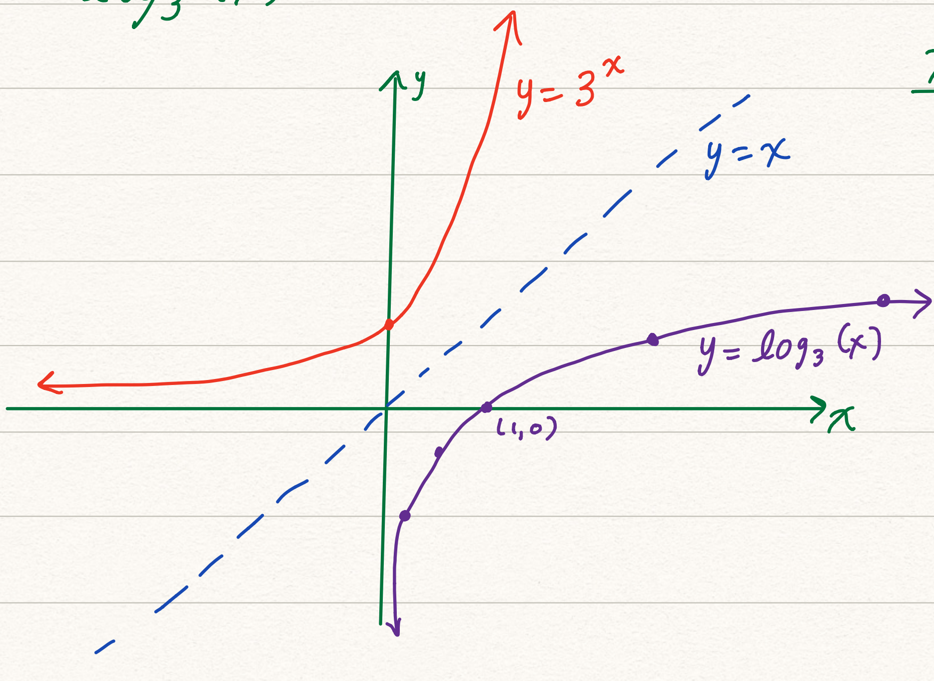
$\rightarrow \ 2 - x > 0$

$2 > x \iff x < 2$

$\rightarrow$ Domain: $(-\infty, 2)$

Graphs of Logarithms

$f(x) = \log_3(x)$



x	y
$1/9$	$\log_3(1/9) = -2$
$1/3$	$\log_3(\frac{1}{3}) = -1$
1	$\log_3(1) = 0$
3	$\log_3(3) = 1$
9	$\log_3(9) = 2$

Two "special" Logarithms:

① The common logarithm

$$f(x) = \log x \overset{def.}{=} \log_{10} x$$

② The natural logarithm

$$f(x) = \ln x \overset{def.}{=} \log_e x$$