

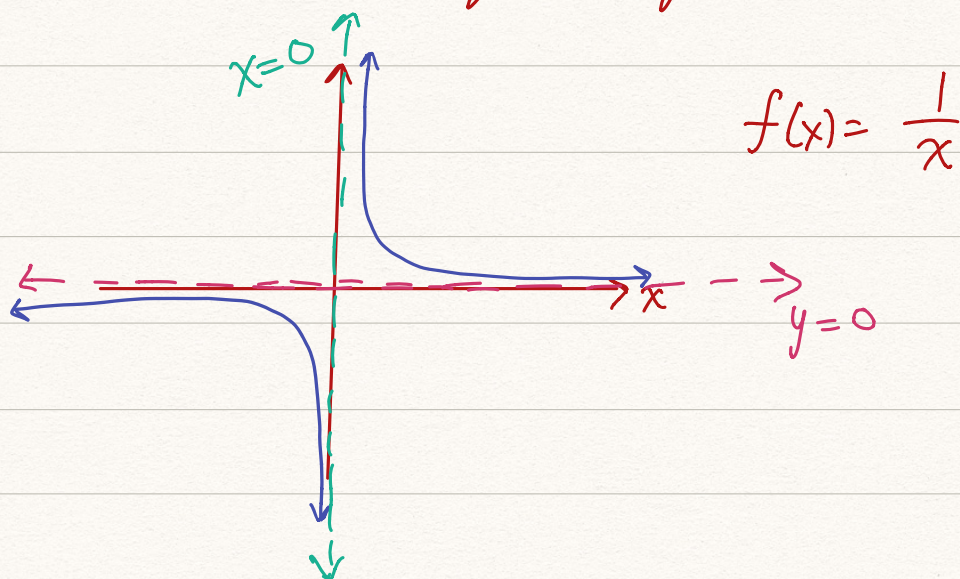
3.6 - Rational Functions

• Looks like: $f(x) = \frac{x^3 + 4x^2 - 7}{x^2 + 1}$

Asymptotes:

• Vertical & Horizontal Asymptotes:

• "Lines that the graph gets close to"



Vertical: Line $x=a$, where "a" makes the denom. = 0.

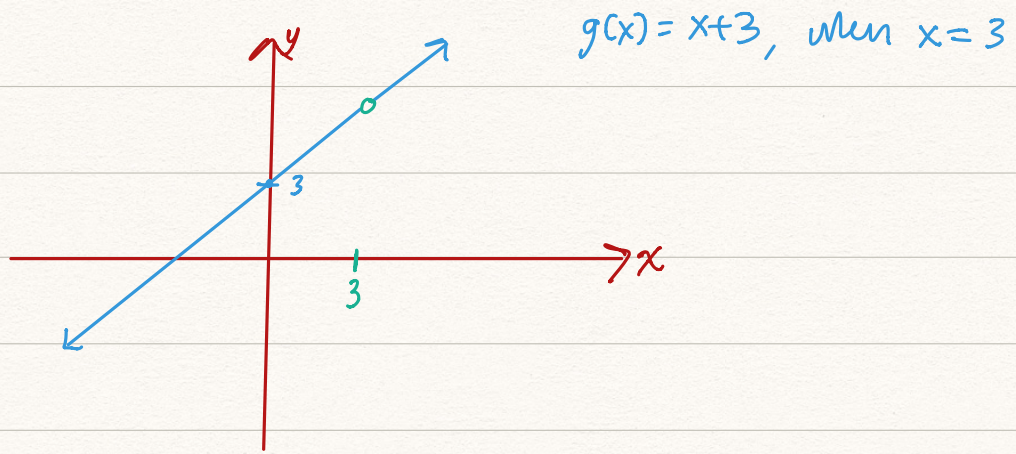
Ex (1) $h(x) = \frac{1}{x-1} \rightarrow$ v.a.: $x=1$

(2) $f(x) = \frac{1}{x^2-9} \rightarrow$ v.a.: $x=3, x=-3$

* Be careful *

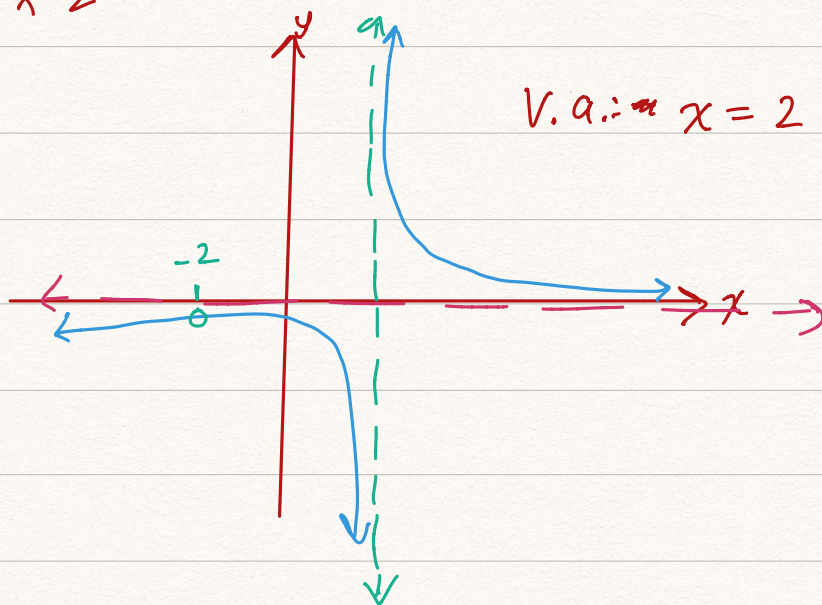
(3) $g(x) = \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3$ assuming $x \neq 3$.

So, g is the same as $x+3$, except for when $x=3$.



④ $f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$ ← assuming $x \neq -2$

So the graph of $f(x)$ is the same as $\frac{1}{x-2}$, except for when $x = -2$.



Horizontal Asymptotes: Depends on leading terms.

BOBO: "Bigger on Bottom, $y=0$ "

Ex: $f(x) = \frac{2x}{x^2+1} \rightarrow$ h.a. $y=0$

BOTN: "Bigger on Top, no h.a."

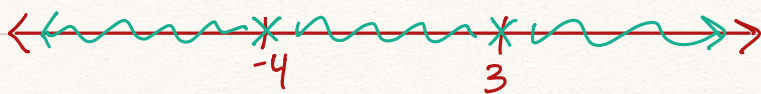
Ex: $f(x) = \frac{3x^2 - 1}{x + 2} \rightarrow$ no h.a.

EATS DC: "Exponents Are The Same, Divide Coeff"

Ex $f(x) = \frac{5x^5 + 1}{3x^5 + x^2 + 2} \rightarrow$ h.a.: $y = \frac{5}{3}$

Ex: Find domain, h.a., & v.a.

(1) $f(x) = \frac{(x+1)(2x-2)}{(x-3)(x+4)} = \frac{2x^2 - 2x + 2x - 2}{x^2 + x - 12}$



Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

h.a: $y = \frac{2}{1} = 2$

v.a: $x = 3, x = -4$