3.4 - The Real Zeros of a Polynomial

Rational Zeros Theorem

Ex: ① Find all rational zeros of $F(x) = 2x^3 + 5x^2 - 4x - 3$

Rational zeros theorem says:

If there are any zeros of F that are rational, they must be of the form

$$\frac{\text{Factor of } -3}{\text{Factor of } 2}$$

Factors of -3: ± 1, ± 3

Factors of 2: ± 1, ± 2

A rational zero of F must be

$$\frac{\pm 1}{1}, \quad \frac{\pm 1}{2}, \quad \frac{\pm 3}{1}, \quad \frac{\pm 3}{2}.$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$1, -1 \qquad \frac{1}{2}, -\frac{1}{2} \qquad 3, -3 \qquad \frac{3}{2}, -\frac{3}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & -4 & -3 \\ & \downarrow & -2 & -3 & 7 \\ \hline & 2 & 3 & -7 & \boxed{4} \end{array}$$

remainder

$$\frac{F(x)}{x-(-1)} = 2x^2 + 3x - 7 + \frac{4}{x-(-1)}$$

- -1 is a zero of $F(x)$ if and only if $(x-(-1))$ is a factor of $F(x)$:

$F(x) = (x-(-1))(\text{some other stuff})$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -4 & -3 \\ & \downarrow & 2 & 7 & 3 \\ \hline & 2 & 7 & 3 & \boxed{0} \end{array}$$ remainder

$$\frac{F(x)}{x-1} = 2x^2+7x+3$$

$$F(x) = (x-1)(2x^2+7x+3)$$
$$F(x) = (x-1)(2x+1)(x+3)$$

$\underline{F(x) \overset{\text{set}}{=} 0:}$

$x-1=0$ $2x+1=0$ $x+3=0$

$x=1$ $x=-\frac{1}{2}$ $x=-3$