

## 1.7 - Equations & Inequalities Including Absolute Values

• Let  $a \geq 0$ , and let  $u$  be an expression. Then

•  $|u| = a$  means:  $u = a$  or  $u = -a$

•  $|u| = -a$  has no solution.

Ex ①  $|2x-3| - 5 = 8$

$$|2x-3| = 13$$

$$2x-3 = 13 \quad \text{or} \quad 2x-3 = -13$$

$$2x = 16$$

$$\boxed{x = 8}$$

$$2x = -10$$

$$\boxed{x = -5}$$

②  $|x+3| = 0$

$$x+3 = 0$$

$$\boxed{x = -3}$$

③  $|6x-3| - 8 = 1$

$$|6x-3| = 9$$

$$6x-3 = 9 \quad \text{or} \quad 6x-3 = -9$$

$$6x = 12$$

$$\boxed{x = 2}$$

$$6x = -6$$

$$\boxed{x = -1}$$

④  $|x-1| = |x+5|$

$$x-1 = x+5 \quad \text{or} \quad x-1 = -(x+5)$$

~~$$-1 = 5$$~~

$$x-1 = -x-5$$

$$2x-1 = -5$$

$$2x = -4$$

$$\boxed{x = -2}$$

$$\textcircled{5} |2x-1| = x+5$$

$$2x-1 = x+5 \quad \text{or} \quad 2x-1 = -(x+5)$$

$$\boxed{x = 6}$$

$$2x-1 = -x-5$$

$$3x = -4$$

$$\boxed{x = -\frac{4}{3}}$$

## Inequalities with Absolute Values

Ex  $\textcircled{1} |4x-1| \leq 9$

$$4x-1 \leq 9 \quad \text{and} \quad 4x-1 \geq -9$$

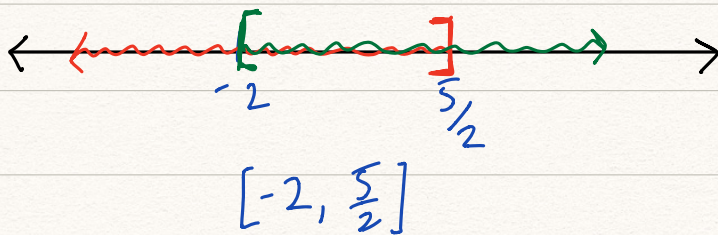
$$4x \leq 10$$

$$4x \geq -8$$

$$\boxed{x \leq \frac{5}{2}}$$

$$\boxed{x \geq -2}$$

since  $\leq$



$$\textcircled{2} |3x+3| - 6 \leq 0$$

$$|3x+3| \leq 6$$

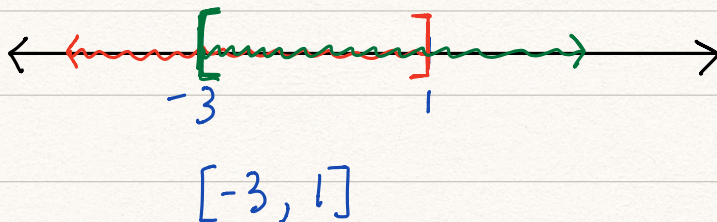
$$3x+3 \leq 6$$

$$\text{and} \quad 3x+3 \geq -6$$

$$\boxed{x \leq 1}$$

$$\boxed{x \geq -3}$$

since  $\leq$



### Steps:

- $\textcircled{1}$  Get absolute value "stuff" by itself on left side.
- $\textcircled{2}$  Break up into and/or statement:
  - $\textcircled{a}$  If  $\geq, >$   
make it **or**
  - $\textcircled{b}$  If  $\leq, <$   
make it **and**
- $\textcircled{3}$  Solve compound inequality.

$$(3) |x-2| < 4|x+4|$$

$$\frac{|a|}{|b|} = \left| \frac{a}{b} \right|$$

$$\frac{|x-2|}{|x+4|} < 4$$

$$\left| \frac{x-2}{x+4} \right| < 4$$

$$\frac{x-2}{x+4} < 4$$

and

since <

$$\frac{x-2}{x+4} > -4$$

$$\frac{x-2}{x+4} - 4 < 0$$

$$\frac{x-2}{x+4} - \frac{4(x+4)}{x+4} < 0$$

$$\frac{x-2-(4x+16)}{x+4} < 0$$

$$\frac{x-2-4x-16}{x+4} < 0$$

$$\frac{-3(x+6)}{x+4}$$

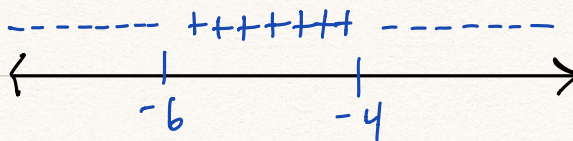
$$\frac{-3x-18}{x+4} < 0$$

$$-3x-18=0$$

$$x+4=0$$

$$x=-6$$

$$x=-4$$



$$(-\infty, -6), (-6, -4), (-4, \infty)$$

Test Point

$$-7$$

Pos./Neg

$$\frac{(-)(-)}{(-)} = (-) \quad \checkmark$$

$$-5$$

$$\frac{(-)(+)}{(-)} = (+) \quad \times$$

0

$$\frac{(-)(+)}{(+)} = (-) \quad \checkmark$$

$$\Rightarrow (-\infty, -6) \cup (-4, \infty).$$

Do the same for  $\frac{x-2}{x+4} > -4$ , then take intersection.