1.6 - Inequalities

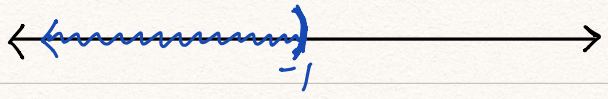
- Solving inequalities is the same as solving equations with one exception:

  *** When we divide or multiply by a negative number, we flip the inequality symbol! ***.

Linear Inequalities

Ex ① $2x < -2$

$x < -1$



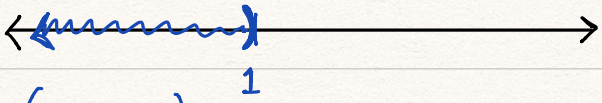
$(-\infty, -1)$

② $7x - 11 < 2(x-3)$

$7x - 11 < 2x - 6$

$5x < 5$

$x < 1$



$(-\infty, 1)$

③ $7(x+2) - 20 - 4x < 3(x-1)$

$7x + 14 - 20 - 4x < 3x - 3$

$3x - 6 < 3x - 3$

$-6 < -3 \longrightarrow$ true! $\longrightarrow (-\infty, \infty)$

④ $2(x+5) + 3x < 5(x-1) + 3$

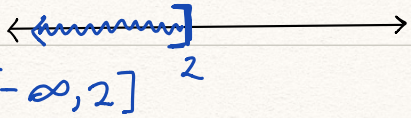
$2x + 10 + 3x < 5x - 5 + 3$

$5x + 10 < 5x - 2$

$10 < -2 \longrightarrow$ false! $\longrightarrow \varnothing$ (no solution)

⑤ $8 - 3x \geq 2$

$-3x \geq -6$ $\longrightarrow$ $x \leq 2$



$(-\infty, 2]$

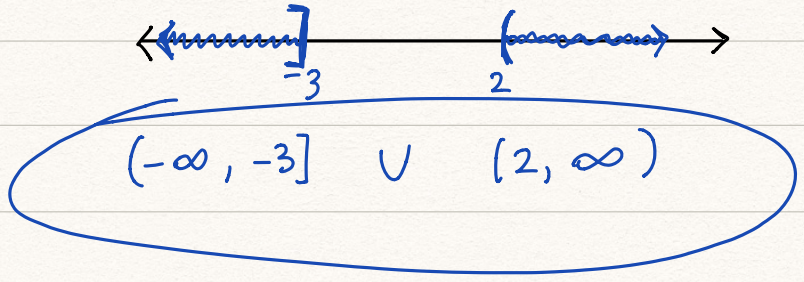
Combining Two Inequalities

Ex: ① $2x+7 \leq 1$ or $3x-2 < 4(x-1)$

$2x \leq -6$ $\qquad$ $3x-2 < 4x-4$

$x \leq -3$ $\qquad$ $2 < x$

$x > 2$



$(-\infty, -3] \cup [2, \infty)$

② $2(x-3)+5 < 9$ and $3(1-x)-2 \leq 7$

$2x-6+5 < 9$ $\qquad$ $3-3x-2 \leq 7$

$2x < 10$ $\qquad$ $-3x \leq 6$

$x < 5$ $\qquad$ $x \geq -2$

$(-\infty, 5) \cap [-2, \infty) = [-2, 5)$

Compound Inequalities

Ex: $-5 < 2x+3 \leq 9$

$-5 < 2x+3$ and $2x+3 \leq 9$

$-8 < 2x$ $\qquad$ $2x \leq 6$

$-4 < x$ $\qquad$ $x \leq 3$

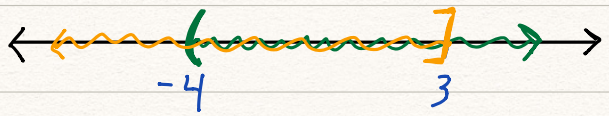
$x > -4$

$-5 < 2x+3 \leq 9$

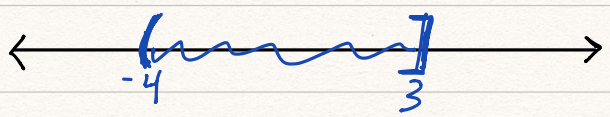
$-5-3 < 2x+3-3 \leq 9-3$

$-8 < 2x \leq 6$

$-4 < x \leq 3$



$(-\infty, 3] \cap (-4, \infty) = (-4, 3]$



$(-4, 3]$

Ex: ① $x^2 + 4x \geq 5x + 6$

$x^2 - x - 6 \geq 0$

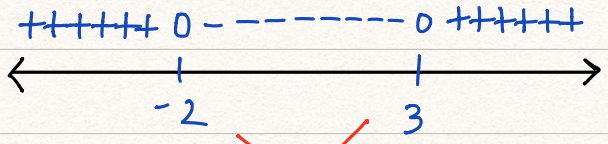
$(x-3)(x+2) \geq 0$

$x - 3 = 0$

$x + 2 = 0$

$x = 3$

$x = -2$



$(-\infty, -2], [-2, 3], [3, \infty)$

Test Points	Pos./Neg.
-3	$(-3-3)(-3+2) = (-6)(-1) = 6$ Pos.
0	$(0-3)(0+2) = (-3)(2) = -6$ Neg.
4	$(4-3)(4+2) = (1)(6) = 6$ Pos.

$\Longrightarrow (-\infty, -2] \cup [3, \infty)$

② $\dfrac{x^2+2x-15}{x-1} \geq 3$

$\dfrac{x^2+2x-15}{x-1} - 3 \geq 0$

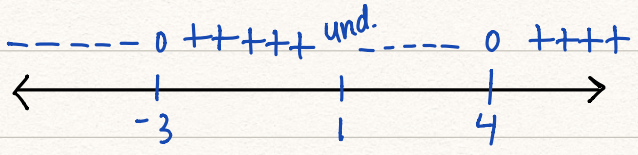
$\vdots$

$\dfrac{x^2-x-12}{x-1} \geq 0$

$\dfrac{(x-4)(x+3)}{x-1} \geq 0$

$x-4=0$
$x+3=0$
$x-1=0$

$x=4$
$x=-3$
$x=1$

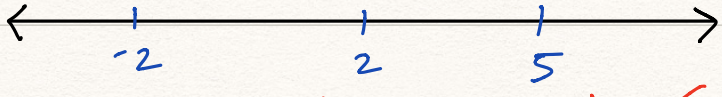


$(-\infty, -3], [-3, 1), (1, 4], [4, \infty)$

$\Longrightarrow [-3, 1) \cup [4, \infty)$.

③ $\dfrac{(x+2)(x-5)}{x-2} < 0$

$x=-2$
$x=5$
$x=2$



$(-\infty, -2), (-2, 2), (2, 5), (5, \infty)$

Test point	Pos/Neg
-3	$\dfrac{(-)(-)}{(-)} = \dfrac{(+)}{(-)} = (-)$ ✓
0	$\dfrac{(+)(-)}{(-)} = \dfrac{(-)}{(-)} = (+)$ ✗
3	$\dfrac{(+)(-)}{(+)} = \dfrac{(-)}{(+)} = (-)$ ✓

(+) (+)

6 $\quad \frac{(+)(+)}{(+)} = (+)$ ~~X~~

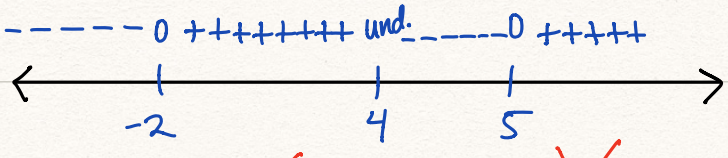
$\Longrightarrow (-\infty, -2) \cup (2, 5)$

(4) $\frac{x^2 - 3x - 10}{x - 4} \leq 0$

$\frac{(x-5)(x+2)}{x-4} \leq 0$

$x - 5 = 0$
$x + 2 = 0$
$x - 4 = 0$

$x = 5$
$x = -2$
$x = 4$



$(-\infty, -2], [-2, 4), (4, 5], [5, \infty)$

$\Longrightarrow (-\infty, -2] \cup (4, 5]$