1.5 - Solving Other Types of Equations

- In 1.1: Learned how to solve linear equations:

$$ax+b=0 \qquad (\text{Ex}: 3x+1=10)$$

- In 1.3: Learned how to solve quadratic equations:

$$ax^2+bx+c=0 \qquad (\text{Ex}: 3x^2-2x=10)$$

(Learned 4 methods for solving these)

- In 1.4: Learned about complex numbers so that we could solve quadratic equations with solutions like $x = 1 \pm \sqrt{-18}$.

- Now, in 1.5: Will learn how to solve "other types" of equations, i.e. equations not of the form

$$ax+b=0 \quad \text{or} \quad ax^2+bx+c=0.$$

Solving by Factoring

Ex ① $x^3 = 2x^2$

$x^3 - 2x^2 = 0$

$x^2(x-2) = 0$

$x^2 = 0$ or $x-2=0$

$x = 0$ $\qquad$ $x = 2$

② $(\sqrt{x})^3 = \sqrt{x}$ $\qquad\qquad$ $\Delta^3 - \Delta = 0$

$(\sqrt{x})^3 - \sqrt{x} = 0$ $\qquad\qquad$ $\Delta(\Delta^2-1) = 0$

$\sqrt{x}\left((\sqrt{x})^2 - 1\right) = 0$

$\sqrt{x}(x-1) = 0 \longrightarrow \sqrt{x} = 0$ or $x-1=0$

$x = 0$ or $x = 1$

③ $x^4 = 27x$

$$x^4 - 27x = 0$$

$$x(x^3 - 27) = 0$$

$x = 0$ or $x^3 = 27$

$$\sqrt[3]{x^3} = \sqrt[3]{27}$$

$$x = 3$$

Solving Rational Equations

- Multiply both sides by the LCD of the denominators.

Ex ① $\dfrac{x+1}{3x-2} = \dfrac{5x-4}{3x+2}$

$$\frac{\cancel{(3x-2)}(3x+2)}{1} \cdot \left(\frac{x+1}{\cancel{3x-2}}\right) = \left(\frac{5x-4}{\cancel{3x+2}}\right) \cdot \frac{(3x-2)\cancel{(3x+2)}}{1}$$

$$(3x+2)(x+1) = (5x-4)(3x-2)$$

$$3x^2 + 3x + 2x + 2 = 15x^2 - 10x - 12x + 8$$

$$3x^2 + 5x + 2 = 15x^2 - 22x + 8$$

$$-12x^2 + 27x - 6 = 0$$

$$x = \frac{-27 \pm \sqrt{(27)^2 - 4(-12)(-6)}}{2(-12)}$$

⋮

$x = \frac{1}{4}$ or $x = 2$

* Check that $x = \frac{1}{4}$ and $x = 2$ work! *

② $\frac{1}{x} + \frac{2}{x+1} = 1$

<u>LCD of $\frac{1}{x}, \frac{2}{x+1}$</u> : $x(x+1)$

$x(x+1)\left(\frac{1}{x} + \frac{2}{x+1}\right) = x(x+1)$

$\cancel{x}(x+1)\frac{1}{\cancel{x}} + x\cancel{(x+1)} \cdot \frac{2}{\cancel{x+1}} = x(x+1)$

$x+1 + 2x = x^2 + x$

$3x+1 = x^2+x$

$0 = x^2 - 2x - 1$

$x^2 - 2x - 1 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+4}}{2}$

$x = 1 \pm \frac{\sqrt{8}}{2}$

$x = 1 \pm \frac{2\sqrt{2}}{2}$

$x = 1 \pm \sqrt{2}$

<u>Check!</u>

$\frac{1}{1+\sqrt{2}} + \frac{1}{1+\sqrt{2}+1} = 1$

$\vdots$

$1 = 1$ ✓

③ $\frac{x}{x-1} - \frac{1}{x+1} = \frac{2x}{x^2-1}$

Since $x^2-1=(x-1)(x+1)$, the LCD is $(x-1)(x+1)$.

$$(x-1)(x+1)\frac{x}{x-1} - (x-1)(x+1)\frac{1}{x+1} = \frac{2x}{x^2-1}\cdot(x+1)(x-1)$$

$$\frac{x\cancel{(x-1)}(x+1)}{\cancel{x-1}} - \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = \frac{2x(\cancel{(x+1)(x-1)})}{\cancel{x^2-1}}$$

$$x^2+x - (x-1) = 2x$$

$$x^2+1 = 2x$$

$$x^2-2x+1=0$$

$$(x-1)(x-1)=0$$

$$x-1=0$$

$$x=1$$

Since $x=1$ "doesn't work" in original equation.

$\rightarrow$ No solution $\varnothing$

Solving Equations w/ Radicals

- Isolate radical, then square (or cube) both sides.

Ex: ① $\sqrt{x-1} = 2$

$(\sqrt{x-1})^2 = (2)^2$

$x - 1 = 4$

$x = 5$

② $\sqrt{x-1} = -2$, $(x \geq 1)$

$(\sqrt{x-1})^2 = (-2)^2$

$x - 1 = 4$

$x = 5$

Check: $\sqrt{5-1} = -2$

$\sqrt{4} = -2$

$2 = -2$ ✗

③ $\sqrt{6y-11} - 2y = -7$

$\sqrt{6y-11} = -7 + 2y$

$6y - 11 = (-7 + 2y)^2$

$6y - 11 = 49 - 28y + 4y^2$

$\vdots$

$y = 6$

④ $\sqrt{2x-5} - \sqrt{x-3} = 1$

$$\sqrt{2x-5} = 1 + \sqrt{x-3}$$

$$2x-5 = 1 + 2\sqrt{x-3} + x - 3$$

$$x-3 = 2\sqrt{x-3}$$

$$\frac{x-3}{2} = \sqrt{x-3}$$

$$\frac{(x-3)^2}{4} = x-3$$

$\vdots$

$$x = -2$$