1.4 - Complex Numbers: Quadratic Eqn's w/ Complex Solutions

Def: The square root of -1 is called i.

$$i = \sqrt{-1}, \quad \text{so} \quad i^2 = -1.$$

So, the equation $x^2 = -1$ has solutions

$$x = \pm\sqrt{-1} = \pm i.$$

- A complex number (in standard form) is a number z of the form $z = a + bi$, where a & b are real numbers.
  - a is called the real part of z
  - b is called the imaginary part of z

- For any positive real number b, we can take the square root of $-b$:

$$\sqrt{-b} = i\sqrt{b}.$$

Careful! The product property for square roots does not work for negative numbers.

$$\cancel{\sqrt{-5}\cdot\sqrt{-2} = \sqrt{(-5)(-2)} = \sqrt{10}}$$

$$\sqrt{-5}\cdot\sqrt{-2} = i\cdot\sqrt{5}\cdot i\cdot\sqrt{2} = i\cdot i\cdot\sqrt{5}\cdot\sqrt{2}$$

$$= i^2\sqrt{10}$$

$$= -\sqrt{10}$$

Ex: Find the real & imaginary parts of the complex numbers

1. $2+5i$
   - real part : 2
   - imag. part : 5

2. $3i$
   - real part : 0
   - imag. part : 3

3. $3+\sqrt{-9} = 3+i\sqrt{9} = 3+3i$
   - real & imag parts both are 3

4. -42
   - real part : -42
   - imag. part : 0

Addition & Subtraction of Complex Numbers

- Similar to combining "like terms" in polynomials.

$$(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$$

Ex ① $(2+5i)+(4-2i) = 6+3i$

② $(2+\sqrt{-9}) - (-2+\sqrt{-4})$

$= (2+3i) - (-2+2i)$ $\qquad \sqrt{-9} = i\sqrt{9} = i\cdot 3 = 3i$

$= (2+3i) + (2-2i)$

$= 4+i$ $\qquad (3i)^2 = 3^2 \cdot i^2 = 9i^2 = -9$

③ $5+(3+7i) = 8+7i$

Multiplication of Complex Numbers

- FOIL!

Ex ① $(2+5i)(4+3i) = \overset{F}{8} + \overset{O}{6i} + \overset{I}{20i} + \overset{L}{15i^2}$

$= 8 + 26i + 15(-1)$

$= -7 + 26i$

② $-2i(5-9i) = -10i + 18i^2$

$= -10i + 18(-1)$

$= -10i - 18$

$= -18 - 10i$

Complex Conjugates & Division

Def: The conjugate of a complex number $z = a+bi$ is the complex number

$\bar{z} = \overline{a+bi} = a-bi.$

• To divide complex numbers, we multiply the numerator & denominator by the conjugate of the denominator.

Ex : ① $\frac{1}{2+i} \cdot \frac{2-i}{2-i} = \frac{2-i}{4-i^2}$ $\quad (x-y)(x+y) = x^2-y^2$

$= \frac{2-i}{4-(-1)}$

$= \frac{2-i}{5}$

$= \frac{2}{5} - \frac{i}{5}$

$= \frac{2}{5} - \frac{1}{5} i$

② $\frac{4+3i}{2-2i} \cdot \frac{2+2i}{2+2i} = \frac{8+8i+6i+6i^2}{4-(2i)^2} = \frac{2+14i}{4-4i^2} = \frac{2+14i}{8}$

$= \frac{2}{8} + \frac{14i}{8}$

$= \frac{1}{4} + \frac{7}{4} i.$

Solving Quadratic Equations

Ex: 1 $x^2+16=0$

$$x^2=-16$$

$$x=\pm\sqrt{-16}$$

$$x=\pm 4i$$

2 $x^2-6x+11=0$

Completing the square:

$$x^2-6x=-11$$

$$x^2-6x+\left(\frac{-6}{2}\right)^2=-11+\left(\frac{-6}{2}\right)^2$$

$$(x-3)^2=-11+9$$

$$(x-3)^2=-2$$

$$x-3=\pm\sqrt{-2}$$

$$x=3\pm i\sqrt{2}$$

Using the quadratic formula:

$$x=\frac{6\pm\sqrt{36-4(1)(11)}}{2(1)}$$

$$=\frac{6\pm\sqrt{-8}}{2}$$

$$=\frac{6}{2}\pm\frac{i\sqrt{8}}{2} \qquad \sqrt{8}=\sqrt{4}\cdot\sqrt{2}=2\sqrt{2}$$

$$=3\pm i\frac{2\sqrt{2}}{2}=3\pm i\sqrt{2}$$