Chapter 1 - Equations & Inequalities

1.1 - Linear Equations in One variable

Def: An equation is a statement that two mathematical expressions are equal.

Ex: $2x+3$ is an expression

$2x+3=7$ is an equation

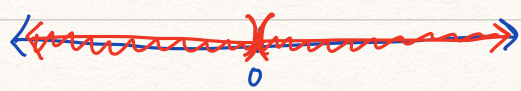
$2x^2-3=14x+3$ is an equation, and $2x^2-3$, $14x+3$ are the expressions.

Def: The domain of the variable in an equation (or in an expression) is the set of all real numbers for which both sides of the equation are defined.

Ex: ① $2x+3$

Domain: all real numbers, or $(-\infty, \infty)$

② $\frac{1}{x}$



Domain: all real numbers except 0.

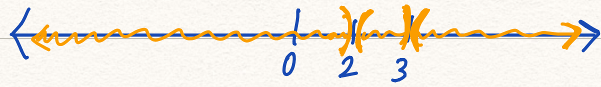
$(-\infty, 0) \cup (0, \infty)$

③ $x-2-\sqrt{x}$

Domain: need $x \geq 0$. So, in interval notation, this is $[0, \infty)$.

④ $\frac{3x}{x^2-5x+6}$

Domain: need $x^2-5x+6 \neq 0$. Factor: $x^2-5x+6=(x-3)(x-2)$



$(-\infty, 2) \cup (2,3) \cup (3, \infty)$

Solving Linear Equations

Steps: ① Simplify both sides

② Move all the terms w/ the variable to one side, and all the constant terms to the other side

③ Isolate the variable completely (usually by dividing).

Ex ① $2x + 1 = 7$

$2x + 1 - 1 = 7 - 1$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

② $6x - (3x - 2(x - 2)) = 11$

$6x - (3x - 2x + 4) = 11$

$6x - (x + 4) = 11$

$6x - x - 4 = 11$

$5x - 4 = 11$

$\quad\ \ +4 \quad +4$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

③ $5x - (3x + 2(x-1)) = 1$

$5x - (3x + 2x - 2) = 1$

$5x - 3x - 2x + 2 = 1$

$2 = 1$ inconsistent!

Note on Equal Signs:

- Equal signs are sacred. They are not "glue" to attach steps together

Example: Evaluate $3x+2$ for $x=-7$.

Bad! $\underbrace{3(-7) = -21 + 2}_{\text{not equal}} = -19$

Good $3(-7) + 2 = -21 + 2 = -19$

Going back to P.6 for a minute:

Rationalizing expressions:

→ "We" don't like radical expressions in denominators.

Ex: ① $\dfrac{\sqrt{5}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{15}}{\sqrt{3 \cdot 3}} = \dfrac{\sqrt{15}}{\sqrt{9}} = \dfrac{\sqrt{15}}{3}$

② $\dfrac{\sqrt[3]{4}}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{\sqrt[3]{12}}{\sqrt[3]{27}} = \dfrac{\sqrt[3]{12}}{3}$

③ The conjugate of an expression $a\sqrt{x} + b\sqrt{y}$ is $a\sqrt{x} - b\sqrt{y}$. When the denominator contains an expression of the form $a\sqrt{x} + b\sqrt{y}$, we mult. the numerator & denominator by its conjugate

$(a-b)(a+b) = a^2 - b^2$

$$\frac{3}{\sqrt{7}+\sqrt{2}} \cdot \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} = \frac{3\sqrt{7}-3\sqrt{2}}{\sqrt{7}^2-\sqrt{2}^2} = \frac{3\sqrt{7}-3\sqrt{2}}{5}$$

$$\frac{x}{3\sqrt{x}-2} \cdot \frac{3\sqrt{x}+2}{3\sqrt{x}+2} = \frac{3x\sqrt{x}+2x}{9x-4} = \frac{3x\,x^{1/2}+2x}{9x-4}$$

$$= \frac{3x^{3/2}+2x}{9x-4}$$

$$= \frac{3\sqrt{x^3}+2x}{9x-4}$$

Solving for a given variable:

① $d = rt$; solve for r.

$$\frac{d}{t} = \frac{rt}{t}$$

$$\frac{d}{t} = r$$

② $A = \frac{(a+b)h}{2}$; solve for h

$$2A = \frac{(a+b)h}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$$

$$2A = (a+b)h$$

$\cancel{2A = ah + bh}$ (Not helpful)

$$\frac{2A}{(a+b)} = \frac{(a+b)h}{(a+b)}$$

$$\frac{2A}{(a+b)} = h$$

③ $T = a + (n-1)d$; solve for d

$$\frac{T-a}{n-1} = \frac{(n-1)d}{(n-1)}$$

$$d = \frac{T-a}{n-1}$$